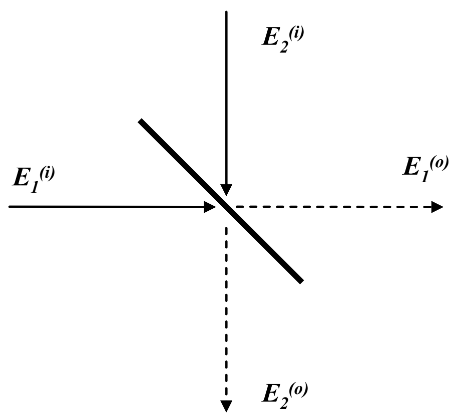
QUANTUM OPTICS
Sommersemester 2008

Blatt 3
zur Übung am 13. Mai 2008

1. The beamsplitter



A classical beamsplitter can be described as a 2×2 matrix that relates the input fields $E_1^{(i)}$, $E_2^{(i)}$ to the output fields $E_1^{(o)}$, $E_2^{(o)}$.

a) Find such a matrix M using the parameter $\sqrt{R}$ where R is the reflection coefficient (for the intensity). Be careful to use the right phase shift (reflection on an optically thicker interface).

b) What is the intensity in the output arms for arbitrary classical input fields?

In the quantum case the same matrix relates the operators $\hat{a}_1^{(i)}$, $\hat{a}_2^{(i)}$ and $\hat{a}_1^{(o)}$, $\hat{a}_2^{(o)}$ and their hermitian conjugates.

c) What is the intensity in the outputs 1 and 2 if a single phtoton enters arm 1 (i.e. the initial state is $|1\rangle_1^{(i)} = \hat{a}_1^{+(i)}|0\rangle$) or arm 2 (i.e. the initial state is $|1\rangle_2^{(i)} = \hat{a}_2^{+(i)}|0\rangle$)?

d) What happens if two identical photons enter arm 1 and arm 2 at the same time (i.e. the initial state is $\hat{a}_1^{+(i)}\hat{a}_2^{+(i)}|0\rangle$)?

e) Show that a coherent state, when entering an input port, is split in a product of two coherent states in the output ports.

2. $G^{(2)}$ function for quantum fields

Consider the electric field operator

$$E^{+}(\mathbf{r}_i, t) = E_0(\hat{a}_{\mathbf{k}} e^{-i(\omega_k t - \mathbf{k}\cdot\mathbf{r}_i)} + \hat{a}_{\mathbf{k}'} e^{-i(\omega_{k'} t - \mathbf{k}'\cdot\mathbf{r}_i)})$$

from two quantum sources S and S'. Assuming equal frequencies ($\nu = c|\mathbf{k}| = c|\mathbf{k}'|$) the second order correlation function $G^{(2)} = \langle E^{-}(\mathbf{r}_1, t)E^{-}(\mathbf{r}_2, t)E^{+}(\mathbf{r}_2, t)E^{+}(\mathbf{r}_1, t)\rangle$ is given by

$$\begin{aligned} G^{(2)} &= E_0^4\langle \hat{a}_{\mathbf{k}}^{+}\hat{a}_{\mathbf{k}}^{+}\hat{a}_{\mathbf{k}}\hat{a}_{\mathbf{k}} + \hat{a}_{\mathbf{k}'}^{+}\hat{a}_{\mathbf{k}'}^{+}\hat{a}_{\mathbf{k}'}\hat{a}_{\mathbf{k}'} \\ &+ \hat{a}_{\mathbf{k}}^{+}\hat{a}_{\mathbf{k}'}^{+}\hat{a}_{\mathbf{k}}\hat{a}_{\mathbf{k}'}[1 + e^{-i(\mathbf{k}-\mathbf{k}')\cdot(\mathbf{r}_1-\mathbf{r}_2)}] \\ &+ \hat{a}_{\mathbf{k}'}^{+}\hat{a}_{\mathbf{k}}^{+}\hat{a}_{\mathbf{k}'}\hat{a}_{\mathbf{k}}[1 + e^{i(\mathbf{k}-\mathbf{k}')\cdot(\mathbf{r}_1-\mathbf{r}_2)}]\rangle \end{aligned}$$

Calculate $G^{(2)}$ for

a) thermal light.

b) laser (poissonian) light.

c) Fock states.

Express the results in terms of the avarage photon number $\langle n\rangle$.