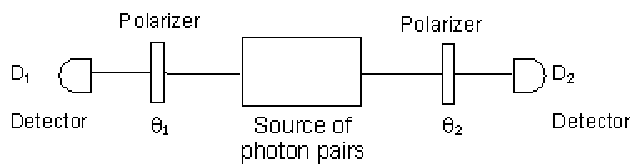
QUANTUM OPTICS
Sommersemester 2008

Blatt 5
zur Übung am 27. Mai 2008

1. Bell's inequality



An entangled state produced by the source of photon pairs has the form:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\big(|1_{1x}, 0_{1y}, 0_{2x}, 1_{2y}\rangle - |0_{1x}, 1_{1y}, 1_{2x}, 0_{2y}\rangle\big),$$

which means that it is either one photon in arm 1 with x-polarization and one in arm 2 with y-polarization or vice versa.

The relation between the annihilation (creation) operators after the polarizer and the operators before the polarizer is:

$$a_j = a_{jx}\cos\theta_j + a_{jy}\sin\theta_j, \qquad j = 1, 2$$

a) What are the probabilities $P_1(\theta_1) = \langle\Psi|a_1^+ a_1|\Psi\rangle$, $P_2(\theta_2) = \langle\Psi|a_2^+ a_2|\Psi\rangle$ to detect a photon after the polarizer 1 and 2, respectively, when the polarizers are set to θ_1, θ_2?

b) What is the joint probability $P_{12}(\theta_1, \theta_2) = \langle\Psi|a_1^+ a_2^+ a_2 a_1|\Psi\rangle$ that one photon is detected after polarizer 1 at angle θ_1 and one after polarizer 2 at angle θ_2?

c) What is the conditional probability $P_{\theta_1}(\theta_2) = P_{12}(\theta_1, \theta_2)/P_1(\theta_1)$ to detect photon 2 in arm 2, given the detection of photon 1 in arm 1? Has the outcome of the measurement in arm 1 any influence on the measurement in arm 2?

d) Calculate the joint probabilities $P(+, \theta_1, +, \theta_2)$ that one photon emerges from polarizer 1 and one photon emerges from polarizer 2, $P(-, \theta_1, -, \theta_2)$ that no photon emerges from either arm and the probabilities $P(+, \theta_1, -, \theta_2)$, $P(-, \theta_1, +, \theta_2)$ that one photon emerges from polarizer 1 and no photon from polarizer 2 and vice versa.

e) In Bell's inequality one considers the correlation

$$C(\theta_1, \theta_2) = P(+, \theta_1, +, \theta_2) + P(-, \theta_1, -, \theta_2) - P(+, \theta_1, -, \theta_2) - P(-, \theta_1, +, \theta_2)$$

and defines the parameter S as

$$S = |C(\theta_1, \theta_2) - C(\theta_1, \theta_2')| + |C(\theta_1', \theta_2) + C(\theta_1', \theta_2')|.$$

Calculate S for the entangled state given above and the polarizer settings $\theta_1 = 0$, $\theta_2 = 3\pi/8$, $\theta_1' = -\pi/4$, and $\theta_2' = \pi/8$.

f) The classical correlation is given by

$$C(\theta_1, \theta_2) = \langle A(\theta_1) B(\theta_2) \rangle = \int A(\theta_1, \lambda) B(\theta_2, \lambda) g(\lambda) d\lambda$$

where $A(\theta_1, \lambda)$, $B(\theta_2, \lambda) \in \{-1, +1\}$, λ is some unknown parameter and $g(\lambda)$ some classical probability distribution. Show that $S \leqq 2$ in the classical case.

(Hint: find inequalities for $|C(\theta_1, \theta_2) - C(\theta_1, \theta_2')| \leqq ?$ and $|C(\theta_1', \theta_2) + C(\theta_1', \theta_2')| \leqq ?$ and then use the fact that $\int g(\lambda) d\lambda = 1$ with a positive definite $g(\lambda)$.