QUANTUM OPTICS
Sommersemester 2008

Blatt 7
zur Übung am 10. Juni 2008

1. Jaynes-Cummings Hamiltonian

The Jaynes-Cummings Hamiltonian is given by

$$H_{JC} = \hbar\omega_0\sigma_z + \hbar\omega a^+a + \hbar g(a\sigma^+ + a^+\sigma^-)$$

a) Show that this couples only states $|e,n\rangle$ and $|g,n+1\rangle$ and that H_{JC} can be written in this basis as

$$H = \hbar\omega\big(n+\frac{1}{2}\big)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \hbar\begin{pmatrix}\delta/2 & g\sqrt{n+1}\\ g\sqrt{n+1} & -\delta/2\end{pmatrix}$$

with $\delta = \omega_0 - \omega$.

b) Find the eigenenergies and show that the eigenstates are

$$\cos\theta_n|e,n\rangle - \sin\theta_n|g,n+1\rangle$$
$$\sin\theta_n|e,n\rangle + \cos\theta_n|g,n+1\rangle$$

with

$$\cos\theta_n = \frac{\Omega_n - \delta/2}{\sqrt{(\Omega_n-\delta/2)^2 + g^2(n+1)}}$$
$$\sin\theta_n = \frac{g\sqrt{n+1}}{\sqrt{(\Omega_n-\delta/2)^2 + g^2(n+1)}}$$

and the generalized Rabi frequency $\Omega_n = \sqrt{(\delta/2)^2 + g^2(n+1)}$.

2. Collapse and revival

If a single atom interacts not with a single fock state $|n\rangle$ but with a coherent state $|\alpha\rangle$ the probability P_e to find the (initially excited) atom in the excited state after some time t is given by

$$P_e(t) = e^{-|\alpha|^2}\sum_n \frac{|\alpha|^{2n}}{n!}\cos^2(g\sqrt{n+1}\cdot t)$$

Plot P_e for coherent fields with mean photon number $\langle n\rangle = 1$, $\langle n\rangle = 4$, $\langle n\rangle = 9$, and $\langle n\rangle = 100$. What is the correlation between revival time and mean photon number?

(Note: Remember $\langle n\rangle = |\alpha|^2$ for coherent states.)