12 Cavity QED

The term cavity QED stands for cavity quantum electrodynamics. It treats the modification of the emission properties of emitters (e.g. atoms) by the modification of the quantum vacuum.

12.1 Spontaneous decay

In previous chapters we have already treated two methods to calculate the spontaneous decay of a two-level atom in free space.

The starting point was usually a Hamiltonian of the form:

$$H = H_{atom} + H_{vacum} + H_{int} \tag{588}$$

$$\frac{1}{2}\hbar\omega_0\sigma_z + \sum_k \hbar\omega a_k^+ a_k + \hbar\sum_k (g_k\sigma^+ a_k + h.c.) \tag{589}$$

Now we will use three different methods to derive the spontaneous emission rate:

12.1.1 Method 1: master equation

This method is the most general one.

It leads to an expression for the time evolution of a small system (e.g. an atom) coupled to a reservoir (e.g. a thermal bath of harmonic oscillators).

For the reduced density matrix ρ_S of the small system we found:

$$\begin{aligned}\frac{\partial}{\partial t}\rho_S = \frac{1}{i\hbar}\left[H_A, \rho_S\right] - \frac{\Gamma}{2}\left(\overline{n}_R + 1\right)\left[\sigma^+\sigma^-\rho_S(t) - \sigma^-\rho_S(t)\sigma^+\right] \\ - \frac{\Gamma}{2}\overline{n}_R\left[\rho_S(t)\sigma^-\sigma^+ - \sigma^+\rho_S(t)\sigma^-\right]\end{aligned} \tag{590}$$

with the rate

$$\Gamma = \frac{\omega_0^3 \mu_{12}^2}{3\pi\varepsilon_0 \hbar c^3} \tag{591}$$

The time evolution of the matrix element

$$\langle e|\rho_S(t)|e\rangle = p_e(t) = p_0 \exp(-\Gamma t) \tag{592}$$

which follows from this equation reveals the irreversible exponential decay.

12.1.2 Method 2: Wigner-Weisskopf theory

In this theory we started with the general state vector

$$|\psi(t)\rangle = c_e(t)e^{-i\omega_0 t}|e,\{0\}\rangle + \sum_k c_{gk}(t)e^{-i\omega_k t}|g,\{1_k\}\rangle \tag{593}$$

which after substitution into the Schroedinger equation gave the following coupled equations:

$$\dot{c}_e(t) = -i\sum_k g_k e^{-i(\omega_k-\omega_0)t} c_{gk}(t) \tag{594}$$

$$\dot{c}_{gk}(t) = -i g_k^* e^{i(\omega_k-\omega_0)t} c_e(t) \tag{595}$$

Integration and iteration resulted finally in the same exponential decay with the same rate as given above.

12.1.3 Method 3: Fermi's golden rule

It is intuitive to use this third method.

The approximation is to set

$$c_e(t) = c_e(0) = 1 \tag{596}$$

in the equation above for $\dot{c}_{gk}(t)$.

Then one finds easily:

$$|c_{gk}(t)|^2 = |g_k|^2 \frac{\sin^2[(\omega_k-\omega_0)t/2]}{(\omega_k-\omega_0)^2/4} \tag{597}$$

The probability for the atom to be in the upper state is

$$P_e = 1 - \sum_k |c_{gk}(t)|^2 \tag{598}$$

$$\approx 1 - \int dk\ |g_k|^2 \frac{\sin^2[(\omega_k - \omega_0)t/2]}{(\omega_k - \omega_0)^2/4} \tag{599}$$

By noting that $g_k = \sqrt{\omega_k/2\hbar\varepsilon_0 V}\,\overrightarrow{\mu}_{12}\widehat{\epsilon}$ and averaging over all possible orientations between the atomic dipole $\overrightarrow{\mu}_{12}$ and the electric field vector $\widehat{\epsilon}$ one can derive:

$$P_e = 1 - \frac{1}{6\varepsilon_0 \pi^2 \hbar c^3} \int d\omega\ \omega^3 |\mu_{12}|^2 \frac{\sin^2[(\omega - \omega_0)t/2]}{(\omega - \omega_0)^2/4} \tag{600}$$

Here we used the mode density of free space in the integral. However, this is not necessarily correct! The key point of cavity QED is:

The spontanoeus emission of an atom is not an intrinsic property, but depends on the atom's environment!

In the equation above for the decay in free space, however, for long enough times (but short enough to justify first order perturbation theory, $c_e(t) = 1$) it is:

$$\lim_{t\to\infty} \frac{\sin^2[(\omega - \omega_0)t/2]}{(\omega - \omega_0)^2/4} = 2\pi\delta(\omega - \omega_0)t \tag{601}$$

and thus

$$\frac{dP_e(t)}{dt} = -\frac{\omega^3|\mu_{12}|^2}{3\varepsilon_0 \pi \hbar c^3} P_e(t) = -\Gamma P_e(t) \tag{602}$$

$$= \left(\frac{2\pi}{\hbar^2}\right) |\langle \mu_{12} E \rangle|^2 D_{free}(\omega) \tag{603}$$

The last row is a form of Fermi's golden rule with the density of states (DOS) $D_{free}(\omega)$ and the interaction $\langle \mu_{12} E \rangle$.

12.2 Spontaneous emission in cavities

A typical cavity QED situation is shown in the following figure.

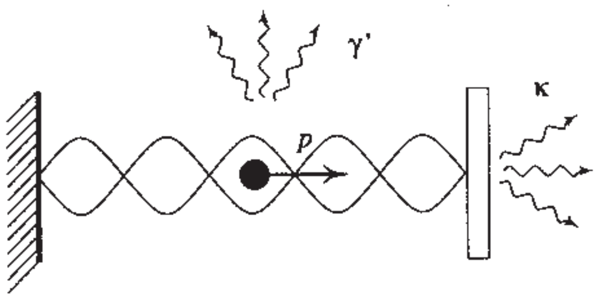

Figure 83: A typical cavity QED system: a single atom is inside a Fabry-Perot cavity [from Meystre in "Sponaneous emission an laser oscillation im microcavities", H. Yokoyama, K. Ujihara, eds.]

A first approach to study the modification of spontaneous emission is to replace the density of states for free space $D_{free}(\omega)$ phenomenologically with a cavity modified density of states $D_{cav}(\omega)$:

$$D_{cav}(\omega) = \frac{\kappa}{2\pi V} \frac{1}{(\kappa/2)^2 + (\omega_{cav} - \omega)^2} \tag{604}$$

with the cavity damping rate κ.

A useful definition for the quality of a cavity is the cavity Q-factor:

$$Q = \omega_{cav}/\kappa \tag{605}$$

With this expression one finds:

- *Enhanced spontaneous emission* for an atom in resonance with the cavity:

$$\Gamma_{cav} = \Gamma_{free} \frac{3Q}{4\pi^2} \left(\frac{\lambda_0^3}{V} \right) \qquad \text{with} \quad \lambda_0 = 2\pi c/\omega \tag{606}$$

- *Suppressed spontaneous emission* for an atom off-resonance with the cavity:

$$\Gamma_{cav} = \Gamma_{free} \frac{3}{16\pi^2 Q} \left(\frac{\lambda_0^3}{V} \right) \qquad \text{with} \quad \lambda_0 = 2\pi c/\omega \tag{607}$$

Enhancement and suppression have been demonstrated for the first time in experiments by Kleppner [D. Kleppner, Phys. Rev. Lett. 47, 233 (1981)] and Haroche [P. Goy, et al., Phys. Rev. Lett. 50, 1903 (1983)].

A more rigorous result is derived via the master equation (at zero temperature):

$$\frac{\partial}{\partial t}\rho = \frac{1}{i\hbar}\left[H_A, \rho\right] - \frac{\Gamma'}{2}\left[\sigma^+\sigma^-\rho(t) + \rho(t)\sigma^+\sigma^- - 2\sigma^-\rho(t)\sigma^+\right] \\ - \frac{\kappa}{2}\left[a^+a\rho(t) + \rho(t)a^+a - 2a\rho(t)a^+\right] \tag{608}$$

where Γ' is the coupling of the atom to a fraction of free space (not covered by the solid angle of the cavity) and κ is the damping rate of the cavity (loss of photons to a reservoir).

It is useful to write this master equation as

$$\frac{\partial}{\partial t}\rho = \frac{1}{i\hbar}\left[H_{eff}, \rho\right] + \Gamma'\sigma^-\rho(t)\sigma^+ + \kappa a\rho(t)a^+ \tag{609}$$

In the subspace $\{|e,0\rangle, |g,1\rangle\}$ this equation of motion is equivalent to a Schroedinger equation with an effective (non-Hermitian) Hamiltonian H_{eff}

$$H_{eff} = H_A - i\hbar\frac{\Gamma'}{2}\sigma^+\sigma^- - i\hbar\frac{\kappa}{2}a^+a \tag{610}$$

acting on the (unnormailzed) one-quantum state

$$|\psi(t)\rangle = c_e(t)e^{i\delta/2}|e,0\rangle + c_g(t)e^{-i\delta/2}|g,1\rangle \tag{611}$$

with $\delta = \omega - \omega_0$:

$$i\hbar\partial_t|\psi(t)\rangle = H_{eff}|\psi(t)\rangle \tag{612}$$

With this one finds

$$\frac{dc_e(t)}{dt} = -\frac{\Gamma'}{2}c_e(t) - igc_g(t) \tag{613}$$

$$\frac{dc_g(t)}{dt} = -(i\delta + \kappa/2)c_g(t) - igc_e(t) \tag{614}$$

In the following we divide two regimes for the approximate solution of this equation.

12.2.1 Weak coupling regime

In this regime we assume $g \ll \kappa, \Gamma'$.

Formally integrating of the equation above yields:

$$c_g(t) = -ig \int_0^t dt' \; c_e(t') e^{-(i\delta + \kappa/2)(t-t')} \tag{615}$$

In the weak coupling regime $c_e(t')$ evolves slowly and can be taken out of the integral.

Then:

$$c_g(t) = \frac{-ig}{i\delta + \kappa/2} c_e(t) \tag{616}$$

Substituting this yields:

$$\frac{dc_e(t)}{dt} = -\left[(\Gamma'/2) + \frac{g^2\,(\kappa/2 - i\delta)}{\delta^2 + \kappa^2/4}\right] c_e(t) \tag{617}$$

Hence:

$$|c_e(t)|^2 = P_e(t) = c_e(0)\exp(-\Gamma_{cav}) \quad \text{with} \tag{618}$$

$$\Gamma_{cav} = \Gamma' + \frac{2g^2}{\kappa}\frac{1}{1 + 2(2\delta/\kappa)^2} \tag{619}$$

Replacing g with the expression for the free space rate Γ yields in resonance ($\delta = 0$):

$$\Gamma_{cav}^{(e)} = \frac{3Q}{4\pi^2}\left(\frac{\lambda_0^3}{V}\right)\Gamma \tag{620}$$

and off-resonance:

$$\Gamma_{cav}^{(s)} = \frac{3}{16\pi^2 Q}\left(\frac{\lambda_0^3}{V}\right)\Gamma \tag{621}$$

These expressions are identical to the results derived above.

12.2.2 Strong coupling regime

The general solution for $c_e(t)$ for arbitrary g, Γ', κ is of the form

$$c_e(t) = c_{e1} e^{\alpha_1 t} + c_{e2} e^{\alpha_2 t} \tag{622}$$

with

$$\alpha_{1,2} = -\frac{1}{2}\left(\frac{\Gamma'}{2} + \frac{\kappa}{2} + i\delta\right) \pm \frac{1}{2}\left[\left(\frac{\Gamma'}{2} + \frac{\kappa}{2} + i\delta\right)^2 - 4g^2\right]^{1/2} \tag{623}$$

with some coefficients c_{e1} and c_{e2}.

If $g \gg \Gamma', \kappa$ it follows

$$\alpha_{1,2} = -\frac{1}{2}\left(\frac{\Gamma'}{2} + \frac{\kappa}{2} + i\delta\right) \pm ig \tag{624}$$

This leads to a damped oscillation of $|c_e(t)|^2$ with the Rabi frequency $\Omega = 2g$ whith a damping constant $(\Gamma' + \kappa)/4$.

The following shows a plot of the excited state probability.

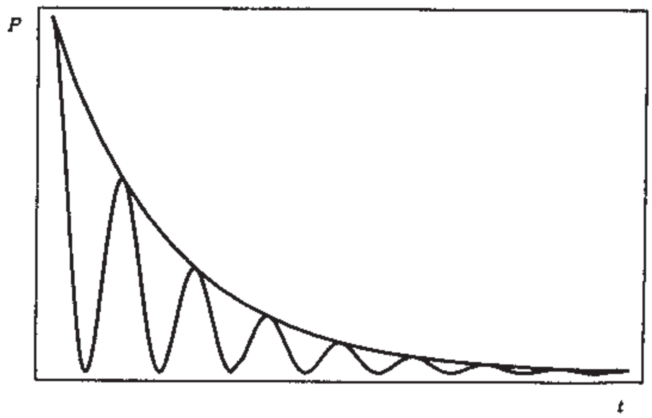

Figure 84: Excited state probabilty in the weak and strong coupling regime [from Meystre in "Sponaneous emission an laser oscillation im microcavities", H. Yokoyama, K. Ujihara, eds.]

The spectrum of the fluorescence light consists of two Lorentzians split by $\Delta\omega = 2g$.

12.3 The spectrum in the strong coupling regime

The appearance of a doublet structure in the fluorescence spectrum in the strong coupling regime is easily interpreted in the dressed atom picture.

The dressed state energies of the combined atom-field system are

$$E_{2n} = \hbar(n + 1/2)\omega_c - \hbar R_n \tag{625}$$

$$E_{1n} = \hbar(n + 1/2)\omega_c + \hbar R_n \tag{626}$$

with the Rabi frequency

$$R_n = \frac{1}{2}\sqrt{\delta^2 + 4g^2(n+1)} \tag{627}$$

The according eigenstates are

$$|2n\rangle = -\sin\vartheta_n\,|en\rangle + \cos\vartheta_n\,|gn+1\rangle \tag{628}$$

$$|1n\rangle = \cos\vartheta_n\,|en\rangle + \sin\vartheta_n\,|gn+1\rangle \tag{629}$$

where

$$\tan 2\vartheta_n = -\frac{2g\sqrt{n+1}}{\delta} \tag{630}$$

The spontaneous decay occurs now from the manifold with one quantum ($|e,0\rangle$, $|g,1\rangle$) to the manifold with zero quantum ($|g,0\rangle$). Obviously, if there is some coupling with non-zero g, the one-quantum manifold splits, but not the zero-quantum manifold.

In case of exact resonance both dressed states of the manifold have equal 'atomic' and 'photonic' weight.

However, if the detuning δ is large it is

$$|10\rangle \approx |e,0\rangle - \frac{g}{\delta}\,|g,1\rangle \tag{631}$$

$$|20\rangle \approx \frac{g}{\delta}\,|e,0\rangle - |g,1\rangle \tag{632}$$

$|10\rangle$ is thus predominantly atomic and $|20\rangle$ predominantly photonic.

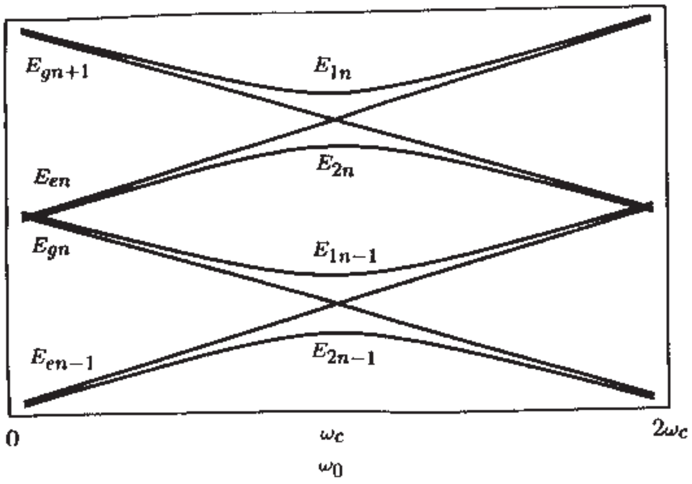

Figure 85: Manifolds of dressed state eigenstates [from Meystre in "Sponaneous emission an laser oscillation im microcavities", H. Yokoyama, K. Ujihara, eds.]

The following figure shows a simulated dressed states spectrum in the strong coupling regime when the detuning is changed.

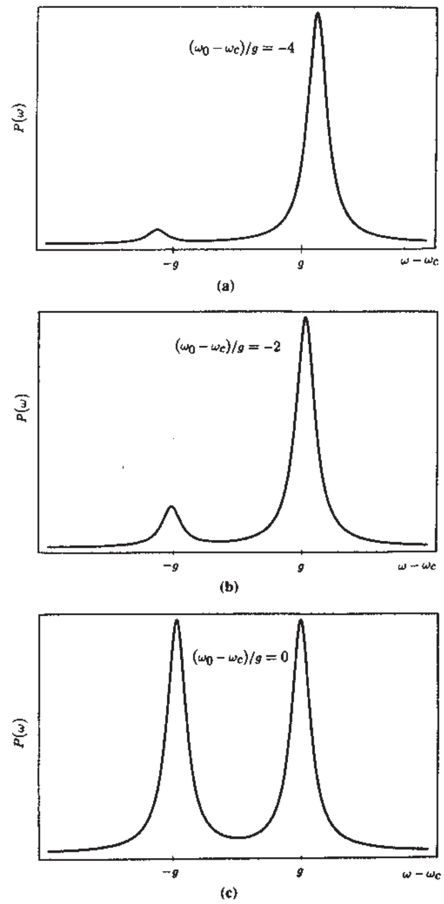

Figure 86: Simulated dressed states spectrum in the strong coupling regime when changing the detuning

The next figure shows an experimental result from an atomic physics experiment performed in the optical domain [R. J. Thompson et al., Phys. Rev. Lett. 68, 1132 (1992)]. Cs atoms from an atomic beam were sent through a high-finesse cavity and the transmission of a probe laser through the cavity was measured.

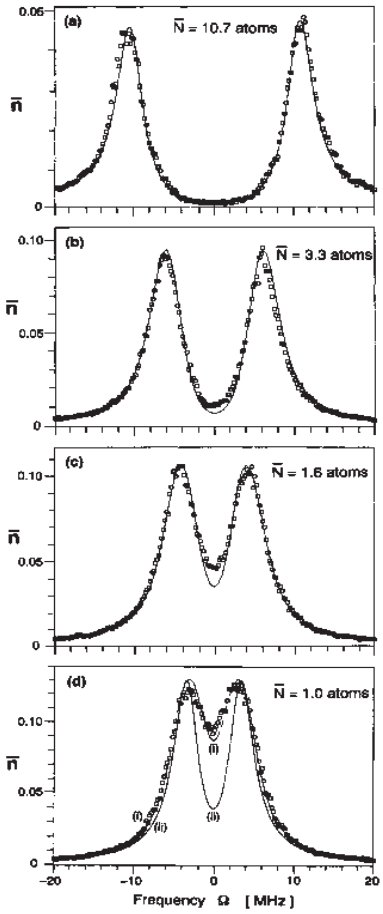

Figure 87: Normal mode splitting of a cavity-atom system for decreasing average number of atoms in the cavity [R. J. Thompson et al., Phys. Rev. Lett. 68, 1132 (1992)].

12.4 Modification of spatial emission pattern

If an emitter is placed inside a cavity not only the total emission rate, but also the emission pattern is changed.

We assume the following geometry of a dipole in a planar cavity formed by two mirrors of reflectivity R_1 and R_2:

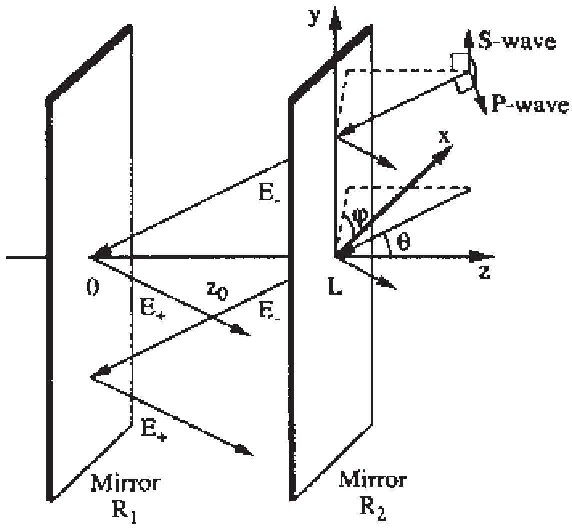

Figure 88: Geometry for the calculation of the electric field inside a Fabry-Perot cavity [from Bjoerk in "Sponaneous emission an laser oscillation im microcavities", H. Yokoyama, K. Ujihara, eds.]

The starting point for the analysis is Fermi's golden rule:

$$\Gamma = \left(\frac{2\pi}{\hbar^2}\right) |\langle \mu_{12} E\rangle|^2 D(\omega) \tag{633}$$

The modification of the rate Γ by the cavity can be interpreted in wo ways:

1. One assumes there is an electric vacuum field of magnitude $E_0 = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}$ in each mode, but the density of states $D(\omega)$ is changed.
2. One assumes a density of states as in free space, but a modified electric field strength. The calculation of the modified field is a classical calculation where

one assumes that a vacuum field of strength E_0 impinges on the cavity and is enhanced or suppressed.

Since E_0 and $D(\omega)$ always enter together, both interpretations are valid.

One can now write the spontaneous emission per unit angle and unit frequency for a y-dipole as:

$$\Gamma(\omega,\vartheta,\varphi) = \left(\frac{2\pi}{\hbar^2}\right) \left| \left\langle \mu_{12} E \right\rangle (\omega,\vartheta,\varphi) \right|^2 \frac{\omega^2 V}{4c^3\pi^3} \tag{634}$$

For s-polarized radiation (E-field perdendicular to the plane of incidence):

$$\Gamma(\omega,\vartheta,\varphi) = \frac{2\pi}{\hbar^2} \frac{\mu_{12}(\omega)\hbar\omega}{2\varepsilon_0 V} \frac{\omega^2 V}{4c^3\pi^3} \cos^2\varphi = \frac{3\Gamma_0 \cos^2\varphi}{8\pi} \tag{635}$$

For π-polarized radiation (E-field parallel to the plane of incidence):

$$\Gamma(\omega,\vartheta,\varphi) = \frac{3\Gamma_0 \sin^2\varphi \cos^2\vartheta}{8\pi} \tag{636}$$

One can check that the total spontaneous emission of a y-dipole is:

$$\Gamma = \frac{3\Gamma_0}{8\pi} \int_0^{2\pi} d\varphi \int_0^{2\pi} d\vartheta \sin\vartheta (\cos^2\varphi + \sin^2\varphi\cos^2\vartheta) = \frac{3\Gamma_0}{4}(1+\frac{1}{3}) = \Gamma_0 \tag{637}$$

Now one only has to calculate the vacuum field entering a cavity.

The field at the position z_0 inside the cavity originating from E_0 from the direction ϑ, φ (and traveling in + and − direction) is:

$$\begin{aligned}
E_- &= E_0\sqrt{T_2}\exp(ik_z(L-z_0))\left[1+\exp(2\pi i)\sqrt{R_1R_2}\exp(2\pi ik_zL)+...\right] \\
&= \frac{\sqrt{1-R_2}\exp(ik(L-z_0)\cos\vartheta)}{1-\sqrt{R_1R_2}\exp(2ik\cos\vartheta)}E_0 && (638) \\
E_+ &= E_0\sqrt{T_2R_1}\exp(ik_z(L+z_0))\left[1+\exp(2\pi i)\sqrt{R_1R_2}\exp(2\pi ik_zL)+...\right] \\
&= \frac{\sqrt{R_1(1-R_2)}\exp(i\left[k(L+z_0)\cos\vartheta+\pi\right])}{1-\sqrt{R_1R_2}\exp(2ik\cos\vartheta)}E_0 && (639)
\end{aligned}$$

where $k_z = k\cos\vartheta = 2\pi\cos\vartheta/\lambda$.

It is now possible to calculate the parallel and z-component of the field for s-polarization

$$|E_{||}|^2 = |E_+ + E_-|^2 \quad \text{and} \quad |E_z|^2 = 0 \tag{640}$$

and for π-polarization:

$$|E_{||}|^2 = |E_+ + E_-|^2\cos^2\vartheta \quad \text{and} \quad |E_z|^2 = |E_+ + E_-|^2\sin^2\vartheta \tag{641}$$

Resonances in the expression occur at $k_z = k\cos\vartheta = m\pi/2$.
The full width half maximum (FWHM) of these resonances in wavelength λ and angle ϑ is:

$$\Delta\lambda = \frac{\lambda^2(1-\sqrt{R_1R_2})}{2\pi L(R_1R_2)^{1/4}} \tag{642}$$

$$\Delta\vartheta = \sqrt{\frac{2\lambda(1-\sqrt{R_1R_2})}{2\pi L(R_1R_2)^{1/4}}} \tag{643}$$

By replacing the electric field field per photon in the expressions for the spontaneous emission with the modified fields an analytical expression for the decay rate of the dipole at an arbitrary position and orientation can be derived.

We will not give these rather lengthy expressions, but show the qualitative behaviour in the following figure:

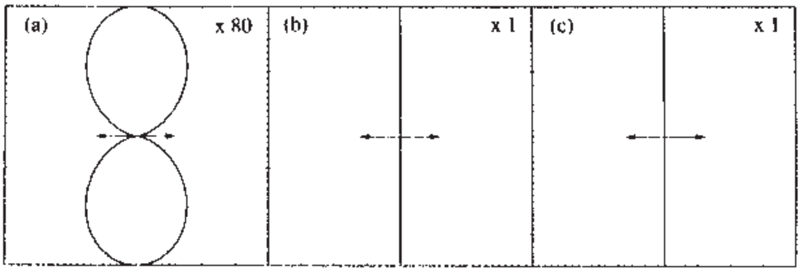

Figure 89: Radiation in the x-z-plane from a z-dipole in free space (a), in a cavity of reflectivity $R = 0.95$ and length 0.5λ (b) and 1.0λ (b) [from Bjoerk in "Sponaneous emission an laser oscillation im microcavities", H. Yokoyama, K. Ujihara, eds.]

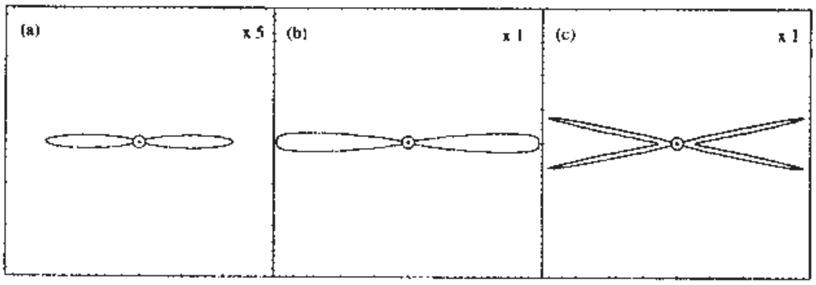

Figure 90: Radiation in the x-z-plane from a y-dipole in a cavity of reflectivity $R = 0.95$ and length 0.49λ (a), 0.5λ (b), and 0.51λ (c) [from Bjoerk in "Sponaneous emission an laser oscillation im microcavities", H. Yokoyama, K. Ujihara, eds.]

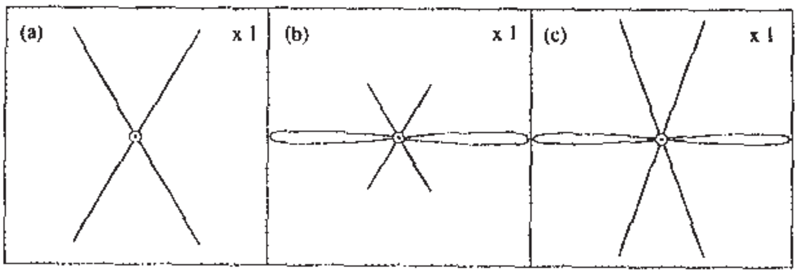

Figure 91: Radiation in the x-z-plane from a y-dipole in a cavity of reflectivity $R = 0.95$ and length 1.0λ (a), 1.0λ (b), and 1.5λ (c). In (b) the dipole is off-centered at $z = \lambda/4$ [from Bjoerk in "Sponaneous emission an laser oscillation im microcavities", H. Yokoyama, K. Ujihara, eds.]

12.5 Modification of energy levels (vacuum shift)

In the discussion of the Wigner-Weisskof theory of spontaneous emission the time evolution of the coefficient $c_e(t)$, i.e. a two-level atom in the excited state and no photon present in the mode was derived:

$$\dot{c}_{e0} = -\frac{1}{6\varepsilon_0\pi^2\hbar c^3}\int d\omega\ \omega^3\mu_{12}\int_0^t dt'\ e^{-i(\omega_k-\omega_0)(t-t')}c_{e0}(t') \tag{644}$$

The evaluation of the last integral gives:

$$\lim_{t\to\infty}\int_0^t dt'\ e^{-i(\omega_k-\omega_0)(t-t')} = \pi\delta(\omega-\omega_0) - P\left[\frac{i}{\omega-\omega_0}\right] \tag{645}$$

It follows:

$$\dot{c}_{e0} = \left(-\frac{\Gamma}{2} + i\Delta\right)c_{e0}(t) \tag{646}$$

A non-vanishing Δ causes a *shift* of the energy level. Experimentally, e.g. the frequency of a transition between two levels of an atom in vacuum is measured. Such an experiment already accounts for the shift. However, if the vacuum is modified, e.g. by placing the atom inside a resonant structure, then the not only the transition rate Γ, but also the shift Δ is modified. It can be detected as a modification of the transition frequency.

First experiments to demonstrate the level shift have been demosntarted in the 80s with single atoms. We follow here the analysis by Hinds and Feld (Phys. Rev. Lett. 59, 2623 (1987)). It starts from expressions for the spontaneous decay rate Γ and the level shift Δ for a two-level atom with transition frequency ω_{12} in a cavity with mode ω_k.

$$\Gamma = \int\int \frac{|\mu_{12}\cdot\hat{\epsilon}|}{\hbar^2}\frac{2\pi\hbar\omega_k}{V}\delta(\omega_{12}-\omega_k)D(\omega_k,\mathbf{k})d\Omega_k d\omega_k \tag{647}$$

$$\Delta = \int\int \frac{|\mu_{12}\cdot\hat{\epsilon}|}{\hbar^2}\frac{2\pi\hbar\omega_k}{V}\frac{1}{\omega 12-\omega_k}D(\omega_k,\mathbf{k})d\Omega_k d\omega_k \tag{648}$$

where μ_{12} is the atomic dipole moment, $\hat{\epsilon}$ describes the polarization of the field, V the quantization volume, and $D(\omega_k,\mathbf{k})$ is the number of modes per unit frequency interval per unit solid angle.

If the atom is inside a cavity formed by a pair of mirrors, then the mode density included by the cavity's solid angle is modified. For these **k**-vector the density is:

$$D_{cav}(\omega_k, \mathbf{k}) = D_{free}\mathcal{L}(\omega_k) \tag{649}$$

where

$$\mathcal{L}(\omega_k) = \frac{(1+F)^{1/2}}{1 + F\sin^2(\omega_k L/c)} \tag{650}$$

with the cavity length L and the parameter $F = 4R/(1-R)^2$ being related to the mirror reflectivity R.

Substituting this results in:

$$\Gamma = \Gamma_{free}\left[1 + (\mathcal{L}(\omega_k) - 1)\, f(\Delta\Omega_{cav}\right] \tag{651}$$

where $f(\Delta\Omega_{cav})$ is the fraction of total free-space spontaneous emission *ordinarly* emitted into the solid angle $\Delta\Omega_{cav}$.

A substitution of this expression gives for the shift Δ of the resonance transition:

$$\Delta = \Gamma_{free}\frac{f(\Delta\Omega_{cav}}{4}\frac{F\sin(2\omega_{12}L/c)}{1 + F\sin^2(2\omega_{12}L/c)} \tag{652}$$

The following figures show setup and results from Hinds and Feld:

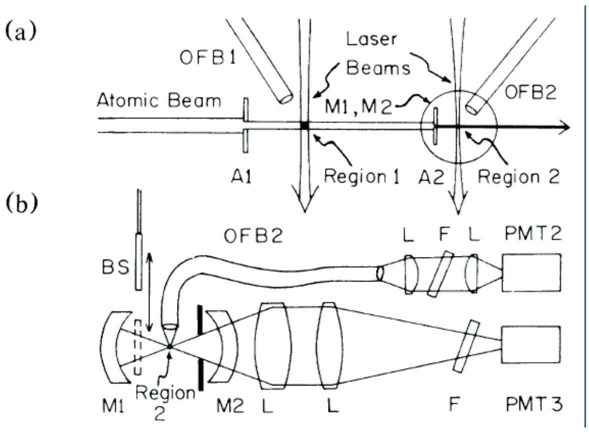

Figure 92: Top (a) and side view (b) of the experiment by Hinds and Feld [PRL 59, 2623 (1987)].

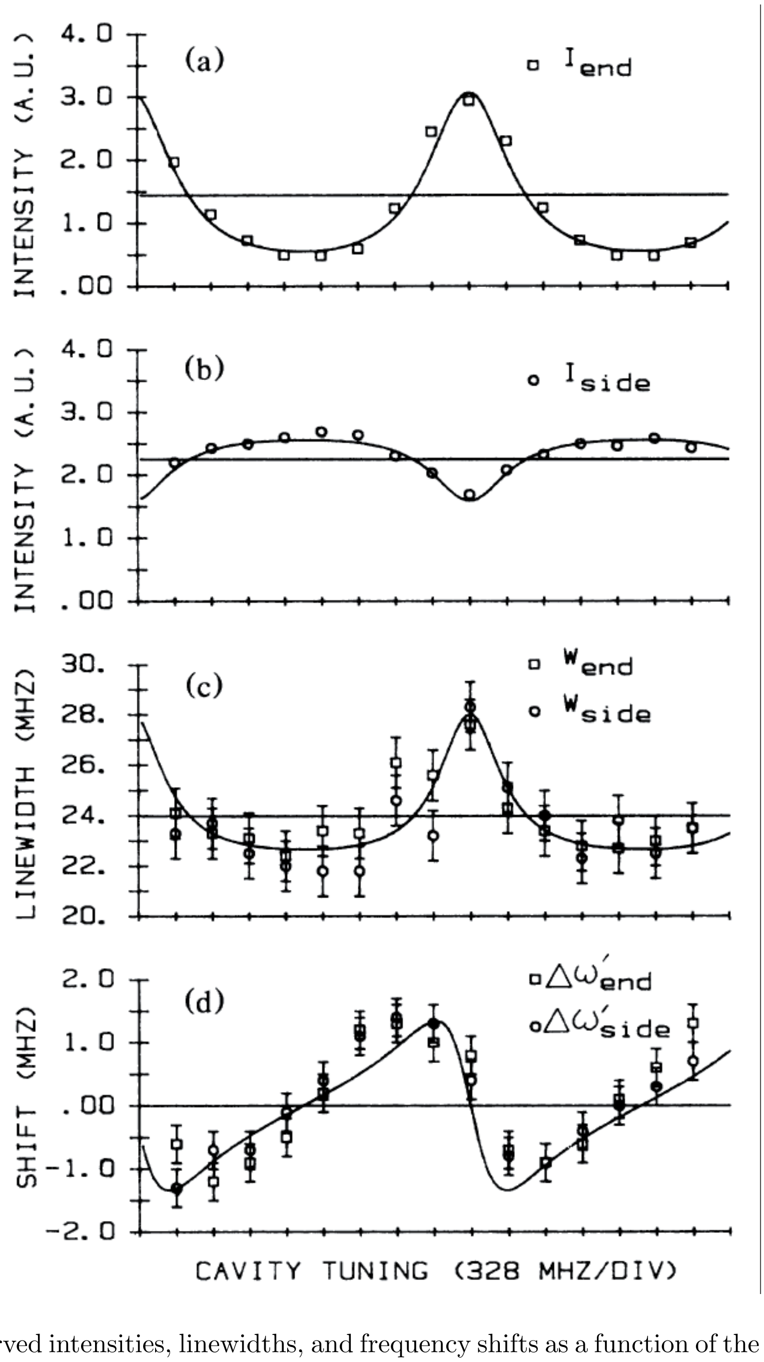

Figure 93: Observed intensities, linewidths, and frequency shifts as a function of the cavity detuning (from bottom to top). The cavity length decreases from left to rigth

12.6 Atoms in cavities

Cavity-QED effects have first been studied in the microwave regime using Rydberg atoms. We have discussed various experiments in previous chapters.

Rydberg atoms provide a very large dipole moment, and thus couple strongly to the electromagnetic field. It is also possible to make superconducting microwave cavities with very large Q-factors. Therefore, the strong coupling regime can be obtained with Rydberg atoms.

With improved optical coating techniques for tiny mirrors cavity QED experiments were performed in the visible (An al., PRL 73, 3375(1995); Rempe, Thompson, Kimble, PRL 68,1132 (1992)).

12.6.1 Generalization of the Jaynes-Cummings-Hamiltonian

In earlier atomic physics experiments in the visible several atoms were interacting with high-Q cavities in ultra-high vacuum chambers. A generalization of the Jaynes-Cummings-Hamiltonian is obtained when several (identical!) atoms are allowed to interact with a single cavity mode(*Tavis-Cummings-Hamiltonian*, Physical Review **170**, 379 (1968)):

$$H_{JC} = \frac{1}{2}\hbar\omega_0 \sum_{l=1}^{N_A} \sigma_l^z + \hbar\omega a^+ a + \hbar \sum_{l=1}^{N_A} g(\vec{r_l})(a\sigma_l^+ + a^+\sigma_l^-) \tag{653}$$

The eigenvalues Λ_p^N and eigenstates $|\psi_{N,p}\rangle$ of this Hamiltonian can be derived analytically (Tavis and Cummings, Physical Review **170**, 379 (1968)).

$$\Lambda_p^N = (A-2p)\,\Omega_N \left[1 - \frac{g^4}{32\Omega_N^4}\,(10p(A-p) - (A-1)(A-2))\right] \tag{654}$$

$$\begin{aligned} |\psi_{N,p}\rangle \;&=\; |N,p\rangle + \tfrac{g^2}{8\Omega_N^2}((A-2p+1)\sqrt{p(A-p+1)})|N,p-1\rangle \\ &\quad -(A-2p-1)\sqrt{(p+1)(A-p)}|N,p\rangle) \end{aligned} \tag{655}$$

where

$$\Omega_N = g\sqrt{N - A/2 + 1/2} \tag{656}$$

is the collective Rabi frequency, A is the number of atoms and N is the total number of excitation. The dressed states can be ordered according to the total number of excitations, i.e. the number of photons or the number of atoms in the system.

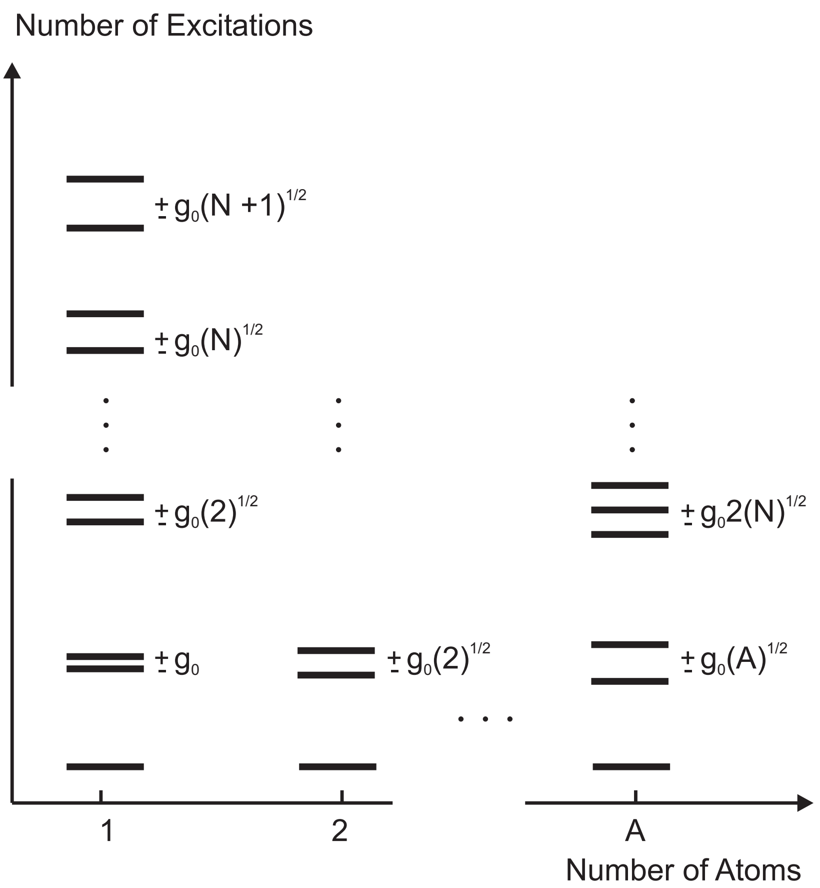

Figure 94: Structure of the eigenstates of the Tavis-Cummings Hamiltonian

12.6.2 Important parameters for Cavity-QED

The most important parameters for the cavity is its Q-factor Q (ratio of resonance frequency and resonance width; determines the field enhancement in the cavity) and its mode volume V_{mode} (the electric field per photon is $\sim 1/\sqrt{V_{mode}}$; determines atom-field coupling strength).

In order to observe cavity QED effects it is important to have both a large Q and a small mode volume V_{mode} or to have a large cavity finesse F

$$F = \frac{FSR}{\Delta f} \sim \frac{c}{L}\frac{Q}{f}$$

with the cavity length L.

A typical setup for a cavity QED experiment is shown in the following figure. It consists of a tunable (piezo control) high-finesse cavity and a beam of (cold) atoms passing through it.

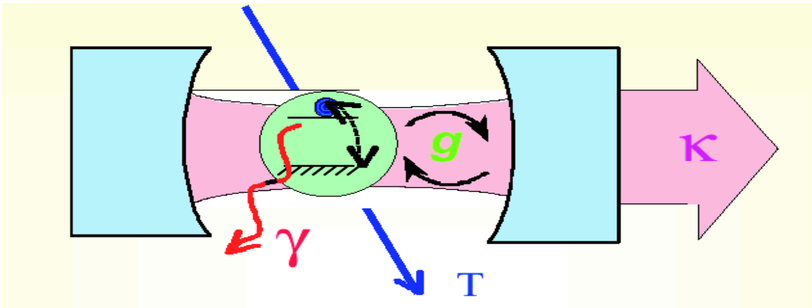

Figure 95: A typical cavity QED experiment with important rates.

There are several rates/times which determine the dynamics of the system:

- The atom-field coupling constant g
- The interaction time T of atoms with the cavity (e.g. given the transition time in an atomic beam experiment)
- The photon damping rate of the cavity κ
- The spontaneous emission rate γ of the atom (by coupling to free field modes)

Two important parameters in cavity QED with atoms are:

- **critical photon number** n_0
  Number of photons required to observe non-linear effects

$$n_0 \approx \frac{\beta^2}{2g^2}$$

  with $\beta = max\{\gamma, 1/T\}$

- **critical atom number** N_0
  Number of atoms in the cavity to observe switching of optical response

$$N_0 \approx \frac{2\beta\kappa}{g^2}$$

  with $\beta = max\{\gamma, 1/T\}$

If both numbers are large it does not matter when the photon number or atom number changes by one. This is the classical limit. In the opposite case, e.g. in the strong coupling regime, both numbers can be much smaller than one ($n_0 \sim 10^{-4}$ and $N_0 \sim 10^{-2}$ have been realized).

In recent experiments (http://www.mpq.mpg.de/qdynamics/index.html, http://www.its.caltech.edu/ qoptics/people.html) ultra-cold atoms from atomic traps are sent into optical cavities in a controlled way. This provides a very long interaction time. Even a mechanical back-action on the cold atoms can be observed.

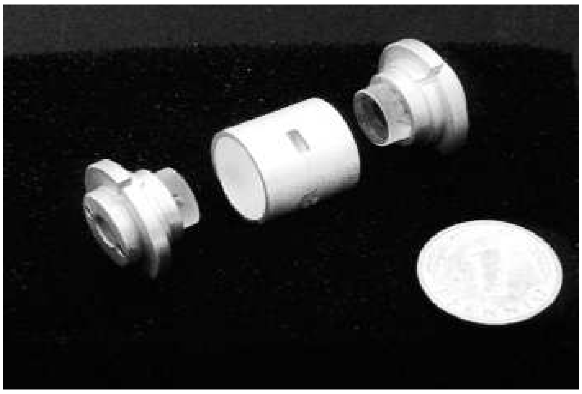

Figure 96: Picture of a high-finess optical cavity

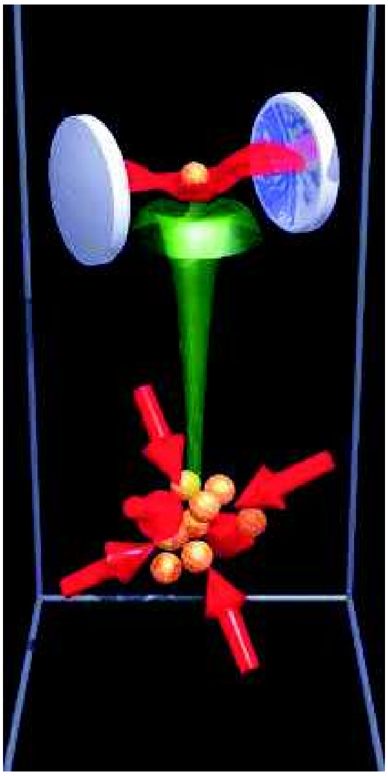

Figure 97: Scheme of a setup where atoms from an atom trap (MOT) are injected into an optical cavity

The following to figures show an experimental setup where the the mode structure of a single neutral atom (Rb) inside an optical cavity is probed. The atom is trapped inside the cavity via a far-detuned trapping laser (dipole trap). The measured transmission of a weak probe laser shows a pronounced double-paek structure, a clear indication of strong coupling.

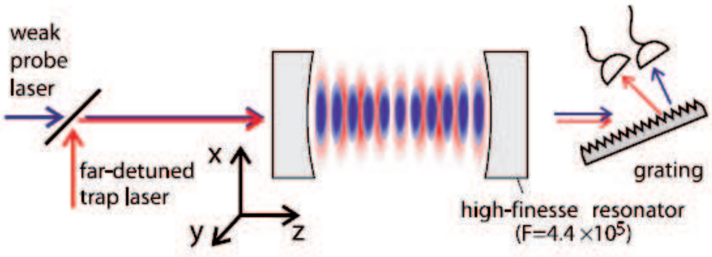

Figure 98: Experimental setup to measure the mode structure of a strongly coupled single atom in an optical cavity via transmission of a probe laser [Maunz et al., Phys. Rev. Lett. 94, 033002 (2005)]

In the strong coupling regime a single atom is capable to block the transmission of a macroscopic pump beam completely!

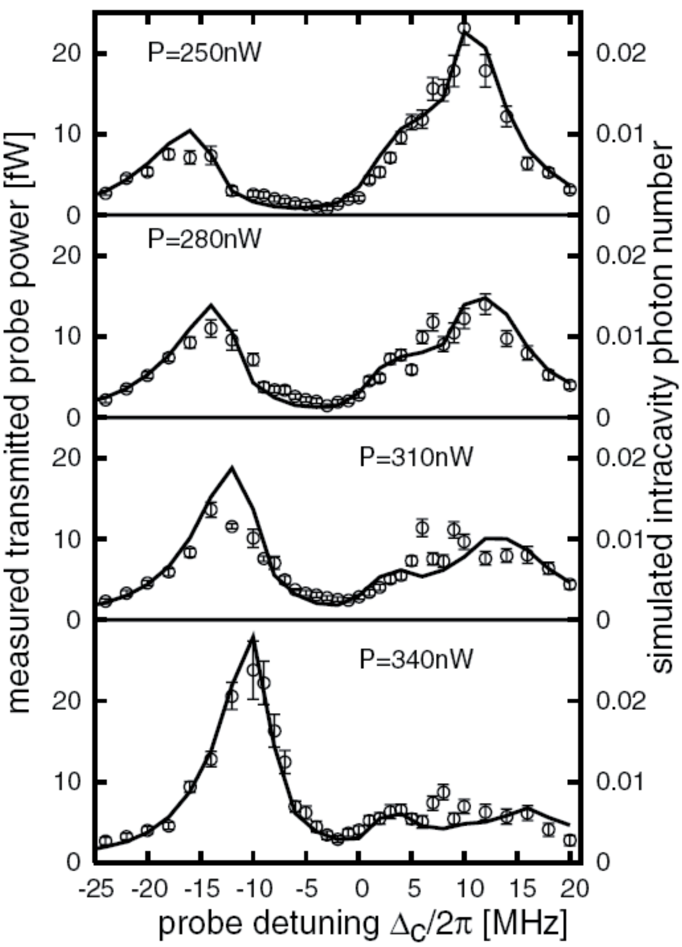

Figure 99: Experimental results from [Maunz et al., Phys. Rev. Lett. 94, 033002]: Transmission of probe laser. The detuning between the cavity and the atom is adjusted by tuning the Stark shift of the atom via the trapping-field power expressed in terms of the transmitted power, P. The average transmission shows well-resolved normal mode peaks. A Monte Carlo simulation (solid lines) describes the data well. (2005)]

12.7 Artificial atoms

Another approach is to use quantum dots (artificial atoms) instead of real atoms. Quantum dots are small semiconductor crystals (several nanometers in size) which are grown epitactically. The crystals form a three-dimensional potential for charge carriers (electrons and holes). Due to the small size of quantum dots the energy levels for both electrons and holes are discrete. Radiative recombination of electrons and holes in quantum dots thus leads to discrete spectral lines similar as in atoms.

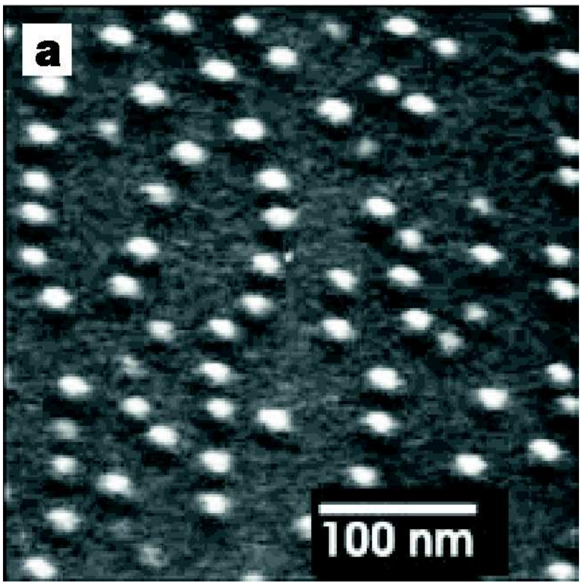

Figure 100: SEM picture of a sample of InAs quantum dots [Reithmeier et al. Nature 432, 197 (2004)]

As the solid state environment provides inferior isolation (compared to UHV vacuum chambers in atomic physics experiments) it is more difficult to obtain the strong coupling regime. However, it is possible to make optical cavities at the fundamental limit of the cavity volume ($(\lambda/2)^3$).

Recently, two groups in Tucson Arizona [T. Yoshie, et al. Nature 432, 200 (2004)] and Würzburg [Reithmeier et al. Nature 432, 197 (2004)] have succeeded to demonstrate the strong coupling regime in all solid-state systems.

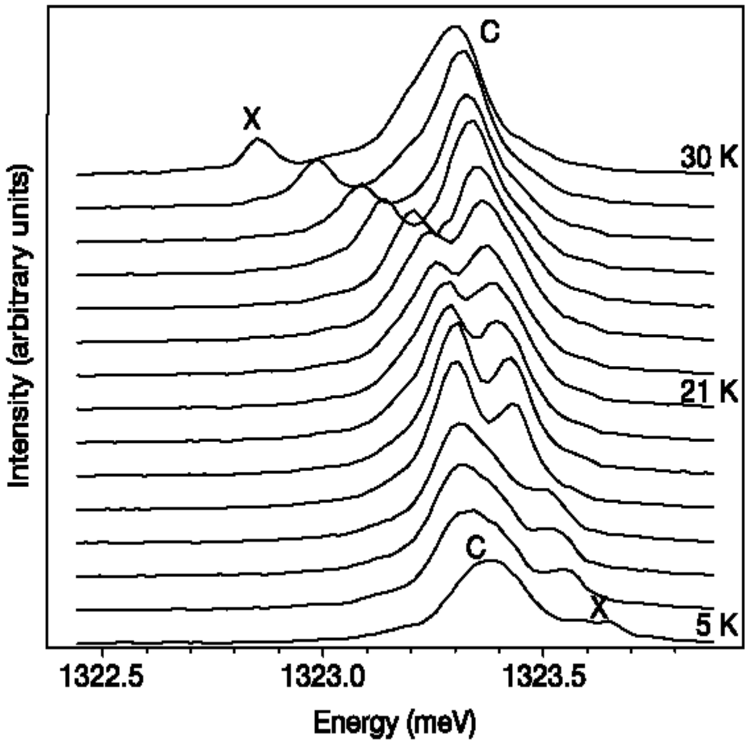

Figure 101: Experimental results from [Reithmeier et al. Nature 432, 197 (2004)]. C denotes the cavity resonance and X the emission line of a single quantum dot

12.8 Examples and applications of CQED-systems

Cavity QED has been driven by fundamental research. However, with the demonstration of cavity QED effects in the visible and even in solid-state systems several applications become possible.

- The enhanced and directed spontaneous emission leads to much more efficient LED's and lasers. A requirement is the fabrication of a laser cavity on the order of the size λ^3 which is possible with today's technology.
- Faster modulation speed of optical devices is obtained by shortening the lifetime of the active material.
- If the fraction of spontaneous emission in a certain mode, the so-called β-factor, approaches unity thresholdless lasers are obtained

- In the strong coupling regime a *coherent* transfer of excitations, i.e. from light to matter or vice versa, is possible. This is of paramount importance for coherent interfaces between light and matter, needed for quantum information processing.
- In the strong coupling regime electro-optic devices based on single photons and single emitters are feasible as already the presence of a single photon or a single atoms causes significant non-linearity.