5 Photon Pairs

5.1 Parametric down-conversion

Parametric down-conversion is a non-linear process, where a wave impinging on a non-linear crystal creates two new light beams obeying energy and momentum conservation:

$$\omega_0 = \omega_1 + \omega_2 \qquad \text{energy conservation} \tag{205}$$

$$k_0 = k_1 + k_2 \qquad \text{momentum conservation (phase-matching)} \tag{206}$$

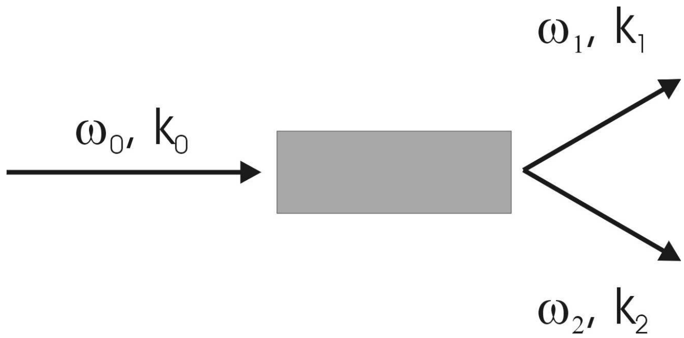

Figure 26: Principle of down-conversion

One of the light beams, e.g., beam 1 is called the *signal* beam, the other, beam 2, is called the *idler* beam.

The Hamiltonian for such a non-linear process is

$$H = \sum_{i=0}^{2} \hbar\omega_i(\widehat{n}_i + 1/2) + \hbar g[a_1^+ a_2^+ a_0 + h.c.] \tag{207}$$

obviously

$$[\widehat{n}_1 + \widehat{n}_2 + 2\widehat{n}_0, H] = 0 \tag{208}$$

which reflects the fission of one photon into two photons.

Usually the pump wave is very strong and can be regarded as a classical field:

$$H = \sum_{i=1}^{2} \hbar\omega_i(\widehat{n}_i + 1/2) + \hbar g[a_1^+ a_2^+ \alpha_0 e^{-i\omega_0 t} + h.c.] \tag{209}$$

with

$$[\widehat{n}_1 - \widehat{n}_2, H] = 0 \tag{210}$$

which means that the two photons are always created together.

The time evolution for the operator $a_1(t)$ can be found using the Heisenberg equation:

$$\dot{a}_1(t) = \frac{1}{i\hbar}[a_1(t), H] \tag{211}$$

$$= -i\omega_1 a_1(t) - ig a_2^+(t)\alpha_0 e^{-i\omega_0 t} \tag{212}$$

With the help of the slowly varying amplitudes

$$A_1(t) = a_1(t)e^{i\omega_1 t} \tag{213}$$

$$A_2(t) = a_2(t)e^{i\omega_2 t} \tag{214}$$

one finds in the case of resonance ($\omega_0 = \omega_1 + \omega_2$):

$$\frac{d}{dt}A_1(t) = -ig\alpha_0 A_2^+(t) \tag{215}$$

$$\frac{d}{dt}A_2(t) = -ig\alpha_0 A_1^+(t) \tag{216}$$

$$\Longrightarrow \tag{217}$$

$$\frac{d^2}{d^2t}A_1(t) = g^2|\alpha_0|^2 A_1^+(t) \tag{218}$$

$$\frac{d^2}{d^2t}A_2(t) = g^2|\alpha_0|^2 A_2^+(t) \tag{219}$$

The general solution is

$$A_1(t) = A_1(0)\cosh(g|\alpha_0|t) - ie^{i\vartheta}A_2^+(0)\sinh(g|\alpha_0|t) \tag{220}$$

$$A_2(t) = A_2(0)\cosh(g|\alpha_0|t) - ie^{i\vartheta}A_1^+(0)\sinh(g|\alpha_0|t) \tag{221}$$

$$\text{where } \alpha_0 = |\alpha_0|e^{i\vartheta} \tag{222}$$

Now it is easy to show that the following holds (with $|\rangle$ denoting the vacuum state):

$$\langle n_1(t)\rangle = \sinh^2(g|\alpha_0|t) = \langle n_2(t)\rangle \qquad \text{and} \tag{223}$$

$$\left\langle : n_1^2(t) : \right\rangle = 2\sinh^4(g|\alpha_0|t) = \left\langle : n_2^2(t) : \right\rangle \tag{224}$$

Thus for the variance:

$$\left\langle \Delta n_1(t)^2 \right\rangle = \left\langle : n_1^2(t) : \right\rangle - \langle n_1(t)\rangle^2 + \langle n_1(t)\rangle \tag{225}$$

$$= \langle n_1(t)\rangle \left[1 + \langle n_1(t)\rangle\right] = \left\langle \Delta n_2(t)^2 \right\rangle \tag{226}$$

With this it is straightforward to evaluate the correlation between signal and idler photon:

$$\sigma_{1,2} = \frac{\langle : \Delta n_1(t)\Delta n_2(t) : \rangle}{\left(\langle \Delta n_1(t)^2\rangle \langle \Delta n_2(t)^2\rangle\right)^{1/2}} = 1 \tag{227}$$

Thus, the signal and idler photons are perfectly correlated: any decrease of photons in the signal requires an equal decrease in the idler beam. This again reflects the fact, that the two photons are always produced together. Whenever a photon is detected in one arm (e.g., idler) there has to be a photon in the other arm (signal). This behavior is utilized in the realization of (non-deterministic or heralded) single photon sources, as depicted in the following picture:

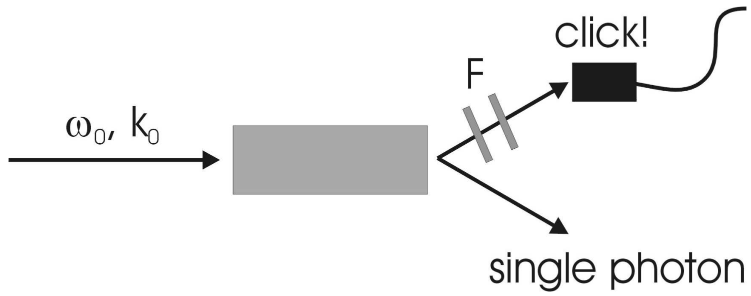

Figure 27: Principle of (non-deterministic) single photon generation. F is a set of filters.

5.2 Non-classical behavior of down-converted light

By using the relation

$$\langle : n_1(t) n_2(t) : \rangle = \left\langle : n_j^2(t) : \right\rangle + \langle n_j(t) \rangle > \left\langle : n_j^2(t) : \right\rangle \qquad \text{for } i = 1,2 \tag{228}$$

it follows from the optical equivalence theorem for normally ordered operator functions ($\left\langle g^{(N)}(a, a^+) \right\rangle = \left\langle g^{(N)}(\alpha, \alpha^*) \right\rangle_P$):

$$\left\langle |\alpha_1|^2 |\alpha_2|^2 \right\rangle_P > \left\langle |\alpha_j|^4 \right\rangle_P = \left(\left\langle |\alpha_1|^4 \right\rangle_P \left\langle |\alpha_2|^4 \right\rangle_P \right)^{1/2} \tag{229}$$

which clearly violates the Schwartz-inequality for classical fields!

Thus, the field generated in the process of down-conversion is inherently non-classical in nature. Presently, the down-conversion process is the main tool for generating non-classical light!

The following pictures show the experiment from Hong, Ou, and Mandel [Phys. Rev. Lett. 59, 2044 (1987)]. A pair of identical photons was created and superimposed on a beamsplitter. When the two photons arrived at the same time they interferred and were either both reflected or transmitted (Photon bunching). Then there was no coincidence count at the detectors.

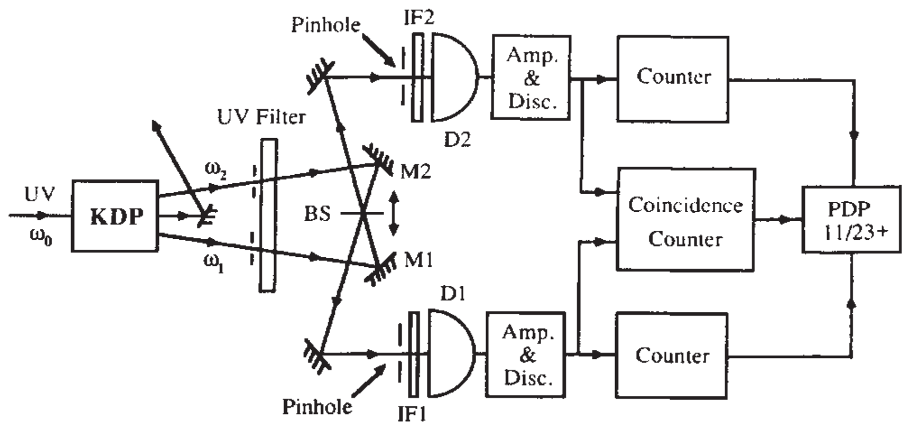

Figure 28: Experimental setup of a photon coincidence experiment using parametric down-conversion [from Hong, Ou, and Mandel, Phys. Rev. Lett. 59, 2044 (1987)]

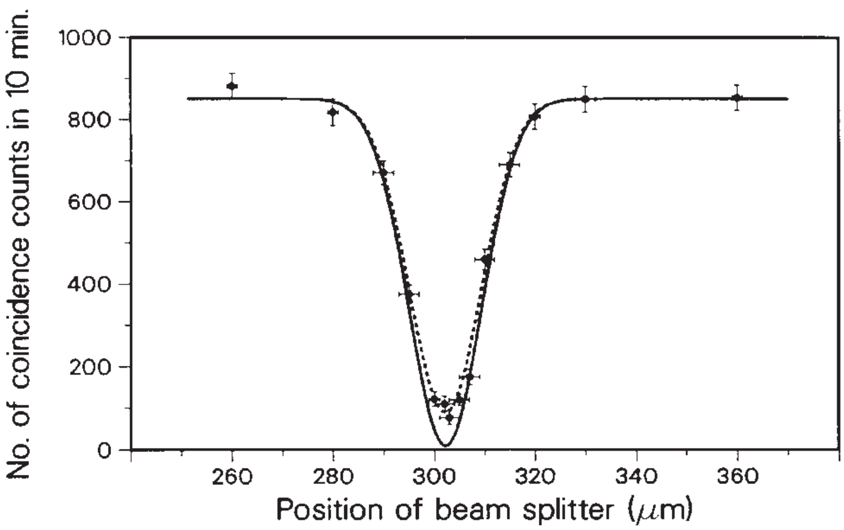

Figure 29: Experimental results (photon coincidence rate versus time delay between the photons) [from Hong, Ou, and Mandel, Phys. Rev. Lett. 59, 2044 (1987)]

5.3 Generation of entangled states

Let us assume the following situation for a down-conversion experiment:

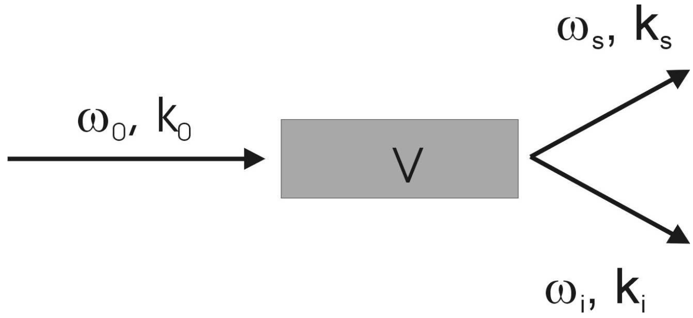

Figure 30: Creation of signal and idler photons in a non-linear crystal of volume V

The interaction Hamiltonian is given by:

$$H_I(t) = \int_V \chi E_0^{(+)} E_i^{(-)}(r,t) E_s^{(-)}(r,t)\ dr \tag{230}$$

where

$$E_0^{(+)} = \alpha_0 e^{i(k_0 r - \omega_0 t)} \qquad \text{classical field} \tag{231}$$

$$E_{i,s}^{(-)} = \alpha_{i,s} e^{-i(k_{i,s} r - \omega_{i,s} t)} a^+_{k_{i,s}} \qquad \text{quantum fields} \tag{232}$$

The parameter χ describes the strengths of the down-conversion process.

Since the signal and idler fields can be in different k-modes we obtain (using ' and " to denote signal and idler modes, respectively):

$$H_I(t) = \tag{233}$$

$$\frac{1}{L^3} \sum_{k',\sigma'} \sum_{k'',\sigma''} \chi \alpha_0 \alpha_{k'\sigma'} \alpha_{k''\sigma''} \int_V dr\ e^{i(k_0 - k' - k'')r} e^{i(\omega_0 - \omega' - \omega'')t} a^+_{k'\sigma'} a^+_{k''\sigma''} + h.c.$$

It is easy to show that it follows for the state $|\psi(t)\rangle$:

$$|\psi(t)\rangle = \exp\left\{ \frac{1}{i\hbar} \int_0^t dt' H_I(t') \right\} |0\rangle \simeq |0\rangle + \tag{234}$$

$$\frac{1}{i\hbar} \frac{1}{L^3} \alpha_0 \sum_{k',\sigma'} \sum_{k'',\sigma''} \chi \alpha_{k'\sigma'} \alpha_{k''\sigma''} \prod_{m=1}^{3} \frac{\sin(\frac{1}{2}\widetilde{k} l_m)}{\frac{1}{2}\widetilde{k} l_m} \frac{\sin(\frac{1}{2}\widetilde{\omega} t)}{\frac{1}{2}\widetilde{\omega} t} |k', \sigma'\rangle |k'', \sigma''\rangle + \tilde{O}(2)$$

here $\widetilde{k} = k_0 - k' - k''$ and $\widetilde{\omega} = \omega_0 - \omega' - \omega''$.

The first fraction in the product accounts for the *phase matching* (momentum conservation), the second fraction for the *energy conservation*.

It is obvious that $|\psi(t)\rangle$ cannot be factorized into single photon states $|k', \sigma'\rangle$.

The state $|\psi(t)\rangle$ is an *entangled state*!

If we fix only two directions k_1 and k_2 and the polarizations σ_1 and σ_2 and assume resonance ($\widetilde{\omega} = 0$) we can find a symmetric state of the form:

$$|\psi(t)\rangle = c_0 |0\rangle + c_2 \left\{ |k_1, \sigma_1\rangle_s |k_2, \sigma_2\rangle_i + |k_2, \sigma_2\rangle_s |k_1, \sigma_1\rangle_i \right\} \tag{235}$$

This can also be denoted as

$$|\psi(t)\rangle = c_0\,|0\rangle + c_2 \left\{ |\omega_s\rangle_{k_1,\sigma_1} |\omega_i\rangle_{k_2,\sigma_2} + |\omega_i\rangle_{k_1,\sigma_1} |\omega_s\rangle_{k_2,\sigma_2} \right\} \tag{236}$$

In the down-conversion process usually $|c_2| \ll |c_0|$ holds.

The two-photon states above reveal the non-local character of quantum mechanics: Let's assume the following setup (the same as above):

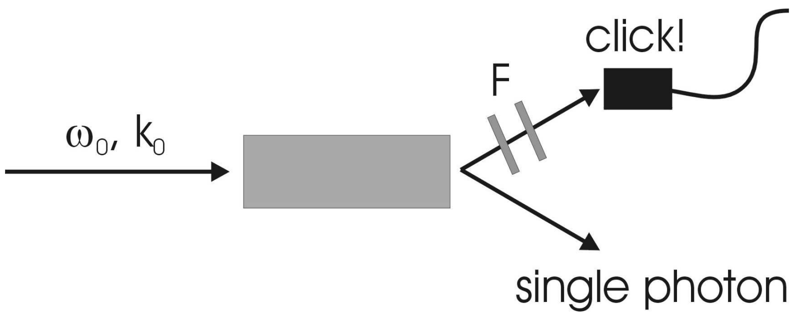

Figure 31: Projection of an entangled state. F is a set of filters.

If we neglect the vacuum contribution we can create a state:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left\{ |\omega_s\rangle_{k_1} |\omega_i\rangle_{k_2} + |\omega_i\rangle_{k_1} |\omega_s\rangle_{k_2} \right\} \tag{237}$$

If we now project the state by measuring a single photon in the k_1-arm at energy ω_s (by choosing an appropriate filter) then the other photon has to have the energy ω_i. But, if we had chosen a filter that transmits ω_i then the photon in the k_2-arm would have had to have the energy ω_s. In each arm no defined energy eigenstate exists. Only after the projection we can assign a definite energy to the photon. The projection occurs immediately, without any time delay.

5.4 Two-photon interference experiments

An interesting experiment with photon pairs, which addresses the issue of interference and indistinguishability was performed by Ou, Wang, and Mandel in 1989 (Phys. Rev. A 40, 1428).
They used the following setup:

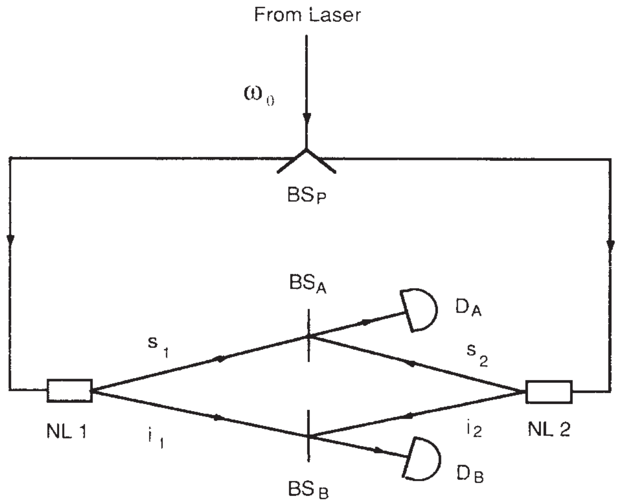

Figure 32: Setup of a two-photon interference experiment [Ou et al., Phys. Rev. A 40, 1428 (1989)]

The results showed an interference, but only an interference of the two-photon coincidence rate!

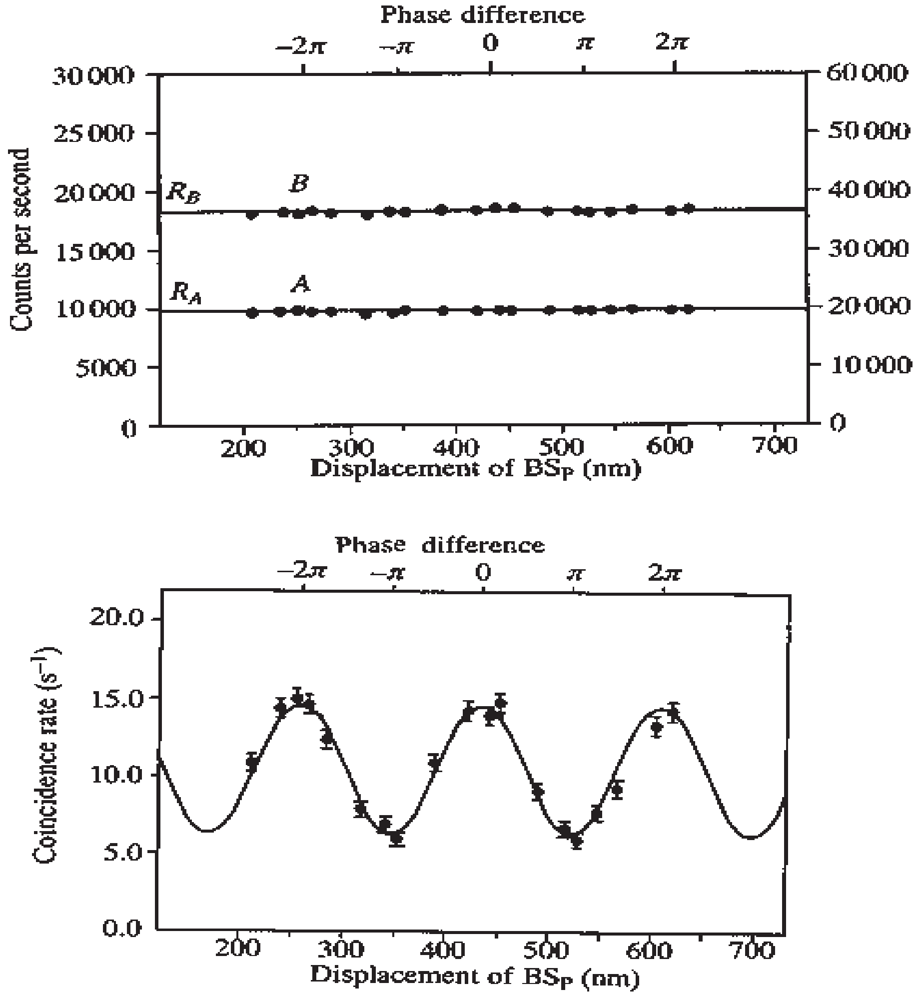

Figure 33: Experimental results of a two-photon interference experiment. Top: single photon rate; bottom: Two-photon coincidence rate [from Ou, et al., Phys. Rev. A 41, 566 (1990)]

The two states after the down-converters NL_1 and NL_2 can be written as:

$$|\psi_1\rangle = c_{0,1}\,|0\rangle_{s1,i1} + \eta_1 A_1\,|\omega\rangle_{s1}\,|\omega'\rangle_{i1} \tag{238}$$
$$|\psi_2\rangle = c_{0,2}\,|0\rangle_{s2,i2} + \eta_2 A_2\,|\omega\rangle_{s2}\,|\omega'\rangle_{i2} \tag{239}$$

where the $c_{0,i}$ are normalization constants, the η_i down-conversion efficiencies and the A_i amplitudes of the classical pump field.

After the beam splitters BS_A and BS_B the field is:

$$E_A^{(+)} = \kappa(a_{s1} + ia_{s2})e^{-i\omega t} \tag{240}$$
$$E_B^{(+)} = \kappa(a_{i1} + ia_{i2})e^{-i\omega t} \tag{241}$$

Then, the count rate in detector A is

$$R_A = \langle\psi_1|\,\langle\psi_2|\,E_A^{(-)}E_A^{(+)}\,|\psi_2\rangle\,|\psi_1\rangle \tag{242}$$
$$= \kappa^2\left[|\eta_1 A_1|^2 + |\eta_2 A_2|^2\right] \tag{243}$$

and a similar expression for detector B. Thus, there is no interference.

However, the two-photon coincidence rate is:

$$R_{AB} = \langle\psi_1|\,\langle\psi_2|\,E_A^{(-)}E_B^{(-)}E_B^{(+)}E_A^{(+)}\,|\psi_2\rangle\,|\psi_1\rangle \tag{244}$$
$$= \kappa^4|c_{01}\eta_1 A_1 - c_{02}\eta_2 A_2|^2 \tag{245}$$
$$\approx \kappa^4\left\{|\eta_1 A_1|^2 + |\eta_2 A_2|^2 - 2|\eta_1 A_1||\eta_2 A_2|\cos\left[\arg A_1 - \arg A_2 + const.\right]\right\} \tag{246}$$

Thus, there is an interference in the two-photon count rate when, e.g., BS_P is moved.

The result can be interpreted in terms of indistinguishability:

1. In two-photon interference there is no way to determine the source of each photon pair. $\Longrightarrow$ interference
2. If, e.g., BS_B is removed (and R_A is measured) detecting a photon in $i1$ means that there was a photon also in $s1$, otherwise the photon detected in D_A was from NL_2. Thus, it is in principle possible to say which path the photon detected in D_A took. $\Longrightarrow$no interference