QUANTUM OPTICS
Wintersemester 2008/2009

Blatt 5
zur Übung am 9. Dezember 2008

Exercise 1: Jaynes-Cummings Hamiltonian

The Jaynes-Cummings Hamiltonian is given by

$$\hat{H}_{JC} = \frac{1}{2}\hbar\omega_0\hat{\sigma}_z + \hbar\omega\hat{a}^\dagger\hat{a} + \hbar g(\hat{a}\hat{\sigma}^+ + \hat{a}^\dagger\hat{\sigma}^-) \tag{1}$$

a) Show that this couples only states $|e,n\rangle$ and $|g,n+1\rangle$ and that H_{JC} can be written in this basis as

$$\hat{H}_{JC} = \hbar\omega(n+\frac{1}{2})\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \hbar\begin{pmatrix}\delta/2 & g\sqrt{n+1}\\ g\sqrt{n+1} & -\delta/2\end{pmatrix}$$

with $\delta = \omega_0 - \omega$.

b) Find the eigenenergies and show that the eigenstates are

$$\cos\theta_n|e,n\rangle - \sin\theta_n|g,n+1\rangle$$
$$\sin\theta_n|e,n\rangle + \cos\theta_n|g,n+1\rangle$$

with

$$\cos\theta_n = \frac{\Omega_n - \delta/2}{\sqrt{(\Omega_n-\delta/2)^2 + g^2(n+1)}}$$
$$\sin\theta_n = \frac{g\sqrt{n+1}}{\sqrt{(\Omega_n-\delta/2)^2 + g^2(n+1)}}$$

and the generalized Rabi frequency $\Omega_n = \sqrt{(\delta/2)^2 + g^2(n+1)}$.

Exercise 2: Jaynes-Cummings Hamiltonian II

Show that in the Jaynes-Cummings model, the operator

$$\hat{X} = \hat{a}^\dagger\hat{a} + \hat{\sigma}^+\hat{\sigma}^-$$

commutes with the Jaynes-Cummings Hamiltonian, equation (1). What is the physical meaning of the operator $\hat{X}$ and of the fact that it commutes with $\hat{H}_{JC}$?

Exercise 3: Collapse and revival

If a single atom interacts not with a single fock state $|n\rangle$ but with a coherent state $|\alpha\rangle$ the probability P_e to find the (initially excited) atom in the excited state after some time t is given by

$$P_e(t) = e^{-|\alpha|^2} \sum_n \frac{|\alpha|^{2n}}{n!} \cos^2(g\sqrt{n+1} \cdot t)$$

Plot P_e for coherent fields with mean photon number $\langle n\rangle = 1$, $\langle n\rangle = 4$, $\langle n\rangle = 9$, and $\langle n\rangle = 100$. What is the correlation between revival time and mean photon number?

(Note: Remember $\langle n\rangle = |\alpha|^2$ for coherent states.)

Exercise 4: Quantum Rabi oscillations

Read *Brune et al.*, PRL **76**, 1800 (1996).

a) What is shown in Fig. 2A–D? Reproduce the solid lines in Fig. 2B–D using the equation given on sheet 5, exercise 2.

b) What is shown in Fig. 2a–d? What frequencies are indicated by the dotted lines? What determines the peak heights at these frequencies?

c) What is shown in Fig. 2α–δ? Plot the solid lines.