QUANTUM OPTICS
Wintersemester 2008/2009

Blatt 6
zur Übung am 13. Januar 2009

Exercise 1: Electromagnetically Induced Transparency (EIT)

In the lecture we considered a 3-level atom in a Λ-configuration, that is pumped by a weak probe beam $E = E_0 \exp(-i\omega_1 t) + c.c.$ and a string pump beam with frequency ω_p having a complex Rabi frequency$\Omega_p \exp(-i\phi)$. We derived the complex susceptibility $\chi = \chi' + i\chi''$, with

$$\chi' = \frac{N_a|\mu_{e,g1}|^2\Delta}{\epsilon_0\hbar Z}\left(\gamma_{g2}(\gamma_e+\gamma_{g2}) + (\Delta^2 - \gamma_e\gamma_{g2} - \Omega_p^2/4)\right) \tag{1}$$

$$\chi'' = \frac{N_a|\mu_{e,g1}|^2}{\epsilon_0\hbar Z}\left(\Delta^2(\gamma_e+\gamma_{g2}) - \gamma_{g2}(\Delta^2 - \gamma_e\gamma_{g2} - \Omega_p^2/4)\right) \tag{2}$$

Here, N_a is the atom number density, γ_i (with $i = e, g1, g2$) are the decay rates of the involved states, $\Delta = (\omega_e - \omega_{g1}) - \omega_1$ is the detuning of the probe field, while $\omega_p = \omega_e - \omega_{g2}$ is assumed. Finally,

$$Z = (\Delta^2 - \gamma_e\gamma_{g2} - \Omega_p^2/4)^2 + \Delta^2(\gamma_e+\gamma_{g2})^2. \tag{3}$$

This tcomplex susceptibility is directly related to the complex index of refraction $n = n' + in''$ over

$$n = \sqrt{1+\chi} \approx 1 + \chi/2 \tag{4}$$

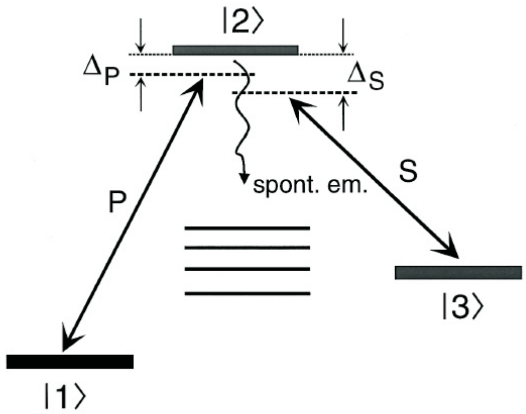
Calculate the absorption coefficient $\alpha = 2n''\omega/c$, the (real part of the) index of refraction n', and the group velocity index $n_g = n' + \omega\frac{dn'}{d\omega}$.

Exercise 2: Electromagnetically Induced Transparency (EIT) II

Plot the the absorption coefficient α and the index of refraction n' from exercise 1 with $\gamma_{g2} = 10^{-4}\gamma_e$ for different pump powers, expressed in terms of $\Omega_p/\gamma_{g1} = 0.1,\ 0.5,\ 1,$ and 2. (You can normalize the expressions by setting $\frac{N_a|\mu_{e,g1}|^2}{\epsilon_0\hbar\gamma_e} = 1$). Compare this with the case of no pump beam incident ($\Omega_p = 0$).

Exercise 3: Stimulated Raman Transitions

Consider again a three level system, as shown in the figure below:



The three states are defined here as $|1\rangle, |2\rangle$, and $|3\rangle$, the detuning of the pump and Stokes laser field are Δ_P and Δ_S, respectively. The coupling strength between the states is given by the Rabi frequencies Ω_P and Ω_S. In the rotating wave approximation, the Hamiltonian can be written as

$$\hat{H}_0 = \hbar(-\Delta_P - \Delta)|1\rangle\langle 1| + \hbar(-\Delta)|2\rangle\langle 2| + \hbar(-\Delta_S - \Delta)|3\rangle\langle 3| \quad (5)$$

$$\hat{H}_I = \frac{\hbar}{2}\Big(\Omega_P(|1\rangle\langle 2| + |2\rangle\langle 1|) + \Omega_S(|3\rangle\langle 2| + |2\rangle\langle 3|)\Big) \quad (6)$$

$$\hat{H} = \hat{H}_0 + \hat{H}_I \quad (7)$$

In the above expression the energy was additionally shifted by $-\hbar\Delta$ with $\Delta = (\Delta_P + \Delta_S)/2$
The state vector in this system can be expanded into

$$|\psi\rangle = c_1(t)|1\rangle + c_2(t)|2\rangle + c_3(t)|3\rangle \quad (8)$$

1. Calculate the equations of motion for the coefficients $c_i(t)$.

2. If both laser fields are similarly detuned ($|\Delta_P - \Delta_S| \ll \Delta$), one can see that $c_2(t)$ carries the fast time dependence at frequencies at the order $\Delta \gg \Gamma$ (where Γ is the decay rate of state 2). If we are interested in timescales slow compared to $1/\Gamma$, we can adiabatically eliminate $c_2(t)$ by making the approximation that it damps to equilibrium instantaneously (i.e. $\partial_t c_2(t) \approx 0$). Show that the equations of motion for the remaining coefficients $c_1(t)$ and $c_3(t)$ under this approximation are:

$$i\hbar\partial_t c_1(t) = \hbar(-\Delta_P - \Delta + \omega_{AC,P})c_1(t) + \frac{\hbar}{2}\Omega_R c_3(t) \quad (9)$$

$$i\hbar\partial_t c_3(t) = \hbar(-\Delta_S - \Delta + \omega_{AC,S})c_3(t) + \frac{\hbar}{2}\Omega_R c_1(t) \quad (10)$$

   where we introduced the Stark shift $\omega_{AC,i} = \Omega_i^2/(4\Delta)$ for $i = P, S$, and the effective Rabi frequency $\Omega_R = \Omega_P\Omega_S/(2\Delta)$.

3. Formulate an effective 2-level Hamiltonian from these equations of motion and discuss the results.