Fundamentals of Optical Sciences

WS 2015/2016 **3 . Exercise** **02.11.2015**

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Prepare your answers for the exercise on 09.11.2015.

Exercise 1

Check whether the retarded potentials of an electric dipole oscillating along the z axis

$$\phi(\vec{r},t) \;=\; \frac{d_0\,\cos\theta}{4\pi\epsilon_0 r}\left(-\,\frac{\omega}{c}\,\sin\left[\omega\left(t-\frac{r}{c}\right)\right] + \frac{1}{r}\,\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\right)$$

and

$$\vec{A}\,(\vec{r},t) \;=\; -\,\frac{\mu_0\,\omega\,d_0}{4\pi r}\,\sin[\,\omega(t-r/c)\,]\,\hat{e}_z \quad ,$$

fulfill the Lorenz gauge condition

$$\mathrm{div}\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t} \;=\; 0$$

not assuming that $r \gg c/\omega$. Here d_0 is the (constant) maximum dipole moment and $\hat{e}_z$ is the unit vector along the z axis.

Exercise 2

One way of making a cheap but good polarizer (except for wavefront distortion) is to use a stack of microscope slides. Assuming an index of refraction $n = 1.52$ and that the slides are all oriented at Brewster's angle with respect to a randomly polarized laser beam, how many slides does it take to attenuate one polarization by a factor of 10^{-4} in intensity compared to the other? Keep in mind that both sides of each slide are at Brewsters angle. Ignore multiple reflections in your calculation.

Exercise 3

Consider a mirror waveguide with an air gap of $d = 2$ μm. What is the cutoff frequency of such a waveguide? How many TE and TM modes does the waveguide support for a wavelength of 780 nm and what are the corresponding bounce angles Θ_m? Derive the mode shape $E_x(y,z)$ of the m-th mode by calculating the interference pattern of the two corresponding propagating waves under the angles $\pm\Theta_m$.

Exercise 4

A radio tower rises to height h above flat horizontal ground. At the top is a magnetic dipole antenna, of radius b, with its axis vertical. FM station *SUN* broadcasts from this antenna at angular frequency ω, with a total radiated power P (that is averaged, of course, over a full radiation period). Neighbors have complained about problems they attribute to excessive radiation from the tower — interference with their stereo systems, garage doors opening and closing mysteriously, and a variety of suspicious medical problems. However, the engineer employed by the city council who measured the radiation level at the base of the tower found it to be well below the accepted standard. You have been hired by the Neighborhood Association to assess the engineer's report.

Magnetic dipole radiation can be described with:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} = \frac{\mu_0 m_0 \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{\phi}}$$
$$\mathbf{B} = \nabla\times\mathbf{A} = -\frac{\mu_0 m_0 \omega^2}{4\pi c^2}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{\boldsymbol{\theta}}.$$

a) In terms of the variables given (not all of which may be relevant, of course), find the formula for the intensity of the radiation at ground level, at distance R from the base of the tower. You may assume that $a \ll c/\omega \ll h$. [*Note*: We are interested only in the *magnitude* of the radiation, not in its *direction* whereas when measurements are taken the detector will be aimed directly at the antenna.]

b) How far from the base of the tower *should* the engineer have made the measurement? What is the formula for the intensity at this location?

c) *SUN*'s actual power output is 35 kW, its frequency is 90 MHz, the antenna's radius is 6 cm and the height of the tower is 200 m. The city's radio-emission limit is 200 μW/cm^2. Is *SUN* in compliance?