Fundamentals of Optical Sciences

WS 2015/2016 **4 . Exercise** **09.11.2015**

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Prepare your answers for the exercise on 16.11.2015.

Exercise 1

a) Show with the help of the (retarded) potentials from the first problem of the 3rd exercise sheet

$$\phi(\vec{r},t) = \frac{d_0 \cos\theta}{4\pi\epsilon_0 r}\left(-\frac{\omega}{c}\sin\left[\omega\left(t-\frac{r}{c}\right)\right] + \frac{1}{r}\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\right)$$

and

$$\vec{A}(\vec{r},t) = -\frac{\mu_0\,\omega\,d_0}{4\pi r}\sin\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{e}_z,$$

and using $r \gg c/\omega$, that the electric and the magnetic fields of a dipole oscillating along the z axis are

$$\vec{E}(\vec{r},t) = \frac{\mu_0\omega^2 d_0}{4\pi r}\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\left(\hat{e}_z - \frac{z}{r}\hat{e}_r\right)$$

and

$$\vec{B}(\vec{r},t) = \frac{1}{c}\left(\hat{e}_r \times \vec{E}\right).$$

Here $\hat{e}_r = \vec{r}/r$ is the unit vector in radial direction.
Hint: Use the relations $\hat{e}_z = \hat{e}_r\cos\theta - \hat{e}_\theta\sin\theta$ and $\hat{e}_r \times \hat{e}_\theta = \hat{e}_\varphi$ to get the form $\vec{E} \propto \hat{e}_\theta$ and $\vec{B} \propto \hat{e}_\varphi$.

b) A rotating dipole $\vec{d}$ can be represented as the superposition of two oscillating dipoles where one dipole oscillates along the x axis and the other one along the y axis. The phase difference between the two dipoles is $\pi/2$, i.e.

$$\vec{d} = d_0\left[\cos(\omega t)\,\hat{e}_x + \sin(\omega t)\,\hat{e}_y\right].$$

Determine the electric and the magnetic fields of an oscillating dipole using the superposition principle and the fields from part a). Calculate also the Poynting vector $\vec{S}$, its time average $\left\langle \vec{S} \right\rangle$, and the emitted power P. Compare P with the emitted power

$$P_1 = \frac{\mu_0\,d_0^2\,\omega^4}{12\,\pi\,c}$$

of an dipole oscillating along one axis. Did you expect your result?
Hint: $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \vec{b}\,(\vec{a}\cdot\vec{c}) - \vec{c}\left(\vec{a}\cdot\vec{b}\right)$; $\hat{e}_r = (x\hat{e}_x + y\hat{e}_y + z\hat{e}_z)/r \Rightarrow \vec{E}\cdot\hat{e}_r = ?$

Exercise 2

Step-index and graded-index optical fibers

a) A step-index fiber has a radius of $a = 5\mu m$, core refractive index $n_1 = 1.45$ and a fractional refractive-index change $\Delta = 0.002$. Determine the shortest wavelength λ_c for which the fiber is a single-mode waveguide. If the wavelength is changed to $\lambda_c/2$, identify the indices (l, m) of all the guided modes.

b) Compare the numerical apertures of the step-index fiber from a) with a graded-index fiber with $n_1 = 1.45$, $\Delta = 0.002$, and a parabolic refractive-index profile ($p = 2$), so that $n^2(y) = n_0^2(1 - p^2y^2)$.

Exercise 3

Asymmetric planar waveguide

Examine the TE field in an asymmetric planar waveguide consisting of a dielectric slab of width d and a refractive index n_1 placed on a substrate of lower refractive index n_2 and covered with a medium of refractive index n_3, where $n_3 < n_2 < n_1$.

a) Determine an expression for the maximum inclination angle θ of plane waves undergoing total internal reflection, and the corresponding numerical aperture (NA) of the waveguide.

b) Write an expression for the self-consistency condition.

c) Determine an approximate expression for the number of modes M (valid when M is very large).

d) For the parameters $n_1 = 1.35$, $n_2 = 1.32$, $n_3 = 1.3$, plot the values of the effective refractive index (β/k_0) as a function of the slab width normalized by the wavelength (d/λ) for the first 3 modes.