Fundamentals of Optical Sciences

WS 2015/2016 **12 . Exercise** **18.01.2016**

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Prepare your answers for the exercise on 25.01.2016.

Exercise 1

Your colleague who is working in the lab next door needs short laser pulses at $\lambda_0 = 850\,\text{nm}$ to do an experiment. You are running a Titan-Sapphire (Ti:Sa) Laser in your lab and consider to send the laser light to his lab through a fiber of 20 m length.

a) In order to tune the working wavelength to 850 nm you take spectra. Estimate the pulse-length Δt based on a laser-spectrum that you have centered at 850 nm and that shows a width of $\Delta\lambda_0 = 20\,\text{nm}$. Further use the assumption that the relation $1/\sqrt{2}{=}\Delta t \Delta\omega$ holds for the pulses produced by your laser system.

b) Use the formula

$$\Delta t_{spread} = \frac{\lambda_0}{c_0} \left| \frac{d^2 n}{d\lambda_0^2} \right| \Delta\lambda_0 \cdot d \tag{1}$$

to compute the additional pulse-spreading Δt_{spread} due to the transmission through the fiber given that the wavelength-dependent refractive index can be described by

$$n^2(\lambda_0) = 1 + \frac{0.6837 \cdot \lambda_0^2}{\lambda_0^2 - 0.0046} + \frac{0.4203 \cdot \lambda_0^2}{\lambda_0^2 - 0.0134} + \frac{0.5850 \cdot \lambda_0^2}{\lambda_0^2 - 64.4933} \tag{2}$$

c) After everything is set-up, it turns out that your colleague needs shorter pulses that are as short as the pulses before they pass the fiber. What can he do to solve this problem on his side of the fiber?

Exercise 2

Calculate the commutator $[\hat{\mathrm{X}}, \hat{\mathrm{Y}}]$ for the quadrature operators

$$\hat{\mathrm{X}} = \frac{\hat{\mathrm{a}} + \hat{\mathrm{a}}^\dagger}{2}$$

and

$$\hat{\mathrm{Y}} = \frac{\hat{\mathrm{a}} - \hat{\mathrm{a}}^\dagger}{2i} \,.$$

Exercise 3

a) Show the validity of the relation

$$\frac{\langle\, \hat{a}^\dagger \hat{a}^\dagger \hat{a}\, \hat{a}\,\rangle}{\langle\, \hat{a}^\dagger \hat{a}\,\rangle^2} \;=\; 1 \;+\; \frac{\Delta n^2 - \bar{n}}{\bar{n}^2}$$

with $\bar{n} := \langle\, n\,\rangle$.

b) Calculate this expectation value

i) for the Fock states $|\, n\,\rangle$

ii) for the coherent states $|\, \alpha\,\rangle$.

Exercise 4

Consider the displacement operator

$$\hat{\mathrm{D}}(\alpha) = \exp[\alpha \hat{a}^\dagger - \alpha^* \hat{a}].$$

a) Show that $\hat{\mathrm{D}}(\alpha)$ is unitary.

b) Which state results if $\hat{\mathrm{D}}(\alpha)$ is applied on the vacuum state $|\, 0\,\rangle$?

Hint: Calculate the action of $\hat{\mathrm{D}}(\alpha)$ either directly or, alternatively, indirectly by using the annihilation operator $\hat{a}$. You will need the Baker-Campbell-Hausdorff (BCH) formula in both cases.

c) Show that

$$\hat{\mathrm{D}}(-\alpha)\; \hat{a}\; \hat{D}(\alpha) = \hat{a} + \alpha\,.$$

Hint: Use the BCH formula.