Examples

This file contains additional information about the examples that are provided with the *CANONICA* package. The differential equations of all examples have been computed with reduze [1, 2].

Contents

1 K4 Integral

This integral topology was first evaluated with the differential equations method in [3]. The topology is given by the following set of propagators

$$\begin{array}{lll} P_1 = (l_1 + l_3)^2, & P_6 = (l_1 + l_2 + l_3 + p_3)^2, & P_{11} = (l_1 + l_2)^2, \\ P_2 = (l_1 + l_2 + p_1 + p_2)^2, & P_7 = l_1^2, & P_{12} = (l_3 + p_1)^2, \\ P_3 = l_3^2, & P_8 = (l_1 + p_1 + p_2)^2, & P_{13} = (l_2 + p_1)^2, \\ P_4 = l_2^2, & P_9 = (l_1 + l_2 + l_3)^2, & P_{14} = (l_1 - p_3)^2, \\ P_5 = (l_1 + p_1)^2, & P_{10} = (l_1 + l_2 + l_3 + p_1 + p_2)^2, & P_{15} = (l_3 - p_3)^2. \end{array} \tag{1}$$

The momenta p_1 and p_2 are incoming and p_3 and p_4 are outgoing. The kinematics are given by

$$p_1^2 = 0, \quad p_2^2 = 0, \quad p_3^2 = 0, \quad p_4^2 = 0, \tag{2}$$

$$p_1 + p_2 = p_3 + p_4, \tag{3}$$

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2. \tag{4}$$

The topology has a basis of 10 master integrals:

$$\begin{aligned} \vec{g}^{\,\mathrm{K4}}(\epsilon, s, t) \;=\; \big(& I_{\mathrm{K4x124}}(1,1,1,1,0,0,0,0,0,0,0,0,0,0,0), \quad I_{\mathrm{K4x1234}}(1,1,1,1,0,0,0,0,0,0,0,0,0,0,0), \\ & I_{\mathrm{K4}}(1,1,1,1,0,0,0,0,0,0,0,0,0,0,0), \quad I_{\mathrm{K4}}(2,1,1,1,1,1,0,0,0,0,0,0,0,0,0), \\ & I_{\mathrm{K4}}(1,1,2,1,1,1,0,0,0,0,0,0,0,0,0), \quad I_{\mathrm{K4}}(1,1,1,1,2,1,0,0,0,0,0,0,0,0,0), \\ & I_{\mathrm{K4}}(1,1,1,1,1,1,-1,0,0,0,0,0,0,0,0), \quad I_{\mathrm{K4}}(1,1,1,1,1,1,0,0,0,0,-1,0,0,0,0), \\ & I_{\mathrm{K4}}(1,1,1,1,1,1,0,0,0,0,0,-1,0,0,0), \quad I_{\mathrm{K4}}(1,1,1,1,1,1,0,0,0,0,0,0,0,0,0) \big). \end{aligned} \tag{5}$$

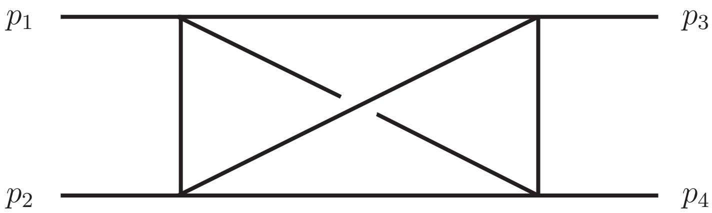

Figure 1: K4 Integral

The mass-dimension is factored out of the master integrals as follows

$$f_i(\epsilon, x) = (t)^{-\dim(g_i)/2} g_i(\epsilon, s, t), \tag{6}$$

with

$$x = \frac{s}{t}. \tag{7}$$

2 Triple box

Analytical results for the triple box integral have first been obtained in [4] and more recently in [5] by using differential equations. The topology is given by the following set of propagators

$$\begin{array}{lll} P_1 = (l_1)^2, & P_6 = (l_3 + p_1 + p_2)^2, & P_{11} = (l_1 + p_3)^2, \\ P_2 = (l_1 + p_1 + p_2)^2, & P_7 = (l_1 + p_1)^2, & P_{12} = (l_2 + p_1)^2, \\ P_3 = l_2^2, & P_8 = (l_1 - l_2)^2, & P_{13} = (l_2 + p_3)^2. \\ P_4 = (l_2 + p_1 + p_2)^2, & P_9 = (l_2 - l_3)^2, & P_{14} = (l_3 + p_1)^2. \\ P_5 = (l_3)^2, & P_{10} = (l_3 + p_3)^2, & P_{15} = (l_1 - l_3)^2. \end{array} \tag{8}$$

The momenta p_1 and p_2 are incoming and p_3 and p_4 are outgoing. The kinematics are given by

$$p_1^2 = 0, \quad p_2^2 = 0, \quad p_3^2 = 0, \quad p_4^2 = 0, \tag{9}$$

$$p_1 + p_2 = p_3 + p_4, \tag{10}$$

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2. \tag{11}$$

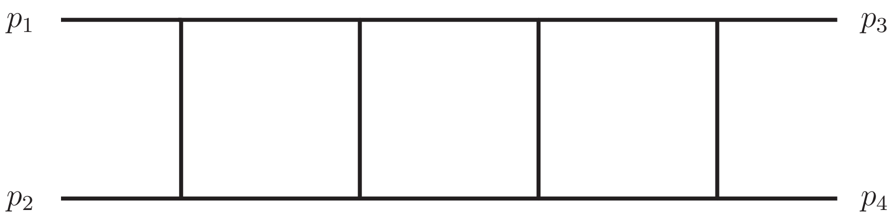

Figure 2: Triple box

The topology has a basis of 26 master integrals:

$$
\begin{aligned}
\vec{g}^{\,\mathrm{TB}}(\epsilon,s,t) \;=\; \big(& I_{\mathrm{TB}}(1,1,1,1,1,1,0,0,0,0,0,0,0,0,0), && I_{\mathrm{TB}}(0,1,1,0,1,1,0,1,0,0,0,0,0,0,0),\\
& I_{\mathrm{TB}}(0,0,1,1,1,1,1,1,0,0,0,0,0,0,0), && I_{\mathrm{TBx124}}(0,1,0,0,1,0,0,1,1,0,0,0,0,0,0),\\
& I_{\mathrm{TB}}(0,1,0,0,1,0,0,1,1,0,0,0,0,0,0), && I_{\mathrm{TB}}(1,0,0,1,1,0,0,1,1,0,0,0,0,0,0),\\
& I_{\mathrm{TB}}(1,1,0,0,1,1,0,1,1,0,0,0,0,0,0), && I_{\mathrm{TB}}(0,0,0,1,1,0,1,1,1,0,0,0,0,0,0),\\
& I_{\mathrm{TB}}(0,0,0,0,1,1,1,1,1,0,0,0,0,0,0), && I_{\mathrm{TB}}(0,1,1,0,1,1,1,1,1,0,0,0,0,0,0),\\
& I_{\mathrm{TB}}(1,1,0,0,0,0,1,1,1,1,0,0,0,0,0), && I_{\mathrm{TB}}(0,1,1,0,0,0,1,1,1,1,0,0,0,0,0),\\
& I_{\mathrm{TB}}(-1,0,1,1,0,0,1,1,1,1,0,0,0,0,0), && I_{\mathrm{TB}}(0,0,1,1,0,0,1,1,1,1,0,0,0,0,0),\\
& I_{\mathrm{TB}}(1,1,1,1,-1,0,1,1,1,1,0,0,0,0,0), && I_{\mathrm{TB}}(1,1,1,1,0,0,1,1,1,1,0,0,0,0,0),\\
& I_{\mathrm{TB}}(0,1,0,0,1,0,1,1,1,1,0,0,0,0,0), && I_{\mathrm{TB}}(1,-1,0,1,1,0,1,1,1,1,0,0,0,0,0),\\
& I_{\mathrm{TB}}(1,0,0,1,1,0,1,1,1,1,0,0,0,0,0), && I_{\mathrm{TB}}(1,1,-1,1,1,0,1,1,1,1,0,0,0,0,0),\\
& I_{\mathrm{TB}}(1,1,0,1,1,0,1,1,1,1,0,0,0,0,0), && I_{\mathrm{TB}}(1,1,-1,0,1,1,1,1,1,1,0,0,0,0,0),\\
& I_{\mathrm{TB}}(1,1,0,0,1,1,1,1,1,1,0,0,0,0,0), && I_{\mathrm{TB}}(1,1,1,1,1,1,1,1,1,1,-1,0,0,0,0),\\
& I_{\mathrm{TB}}(1,1,1,1,1,1,1,1,1,1,0,-1,0,0,0), && I_{\mathrm{TB}}(1,1,1,1,1,1,1,1,1,1,0,0,0,0,0)\big).
\end{aligned} \tag{12}
$$

The mass-dimension is factored out of the master integrals as follows

$$f_i(\epsilon,x) = (t)^{-\dim(g_i)/2} g_i(\epsilon,s,t), \tag{13}$$

with

$$x = \frac{s}{t}. \tag{14}$$

3 Drell-Yan with one internal mass

This topology has been calculated with the differential equations approach in [6]. The topology is given by the following set of propagators

$$
\begin{array}{lll}
P_1 = l_1^2, & P_4 = (l_2 - p_3 - p_4)^2, & P_7 = (l_1 - l_2)^2,\\
P_2 = (l_1 - p_3)^2, & P_5 = (l_2 - p_1)^2, & P_8 = (l_2 - p_3)^2,\\
P_3 = (l_1 - p_3 - p_4)^2 - m^2, & P_6 = l_2^2, & P_9 = (l_1 - p_1)^2.
\end{array} \tag{15}
$$

The momenta p_1 and p_2 are incoming and p_3 and p_4 are outgoing. The kinematics are given by

$$p_1^2 = 0, \quad p_2^2 = 0, \quad p_3^2 = 0, \quad p_4^2 = 0, \tag{16}$$

$$p_1 + p_2 = p_3 + p_4, \tag{17}$$

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2. \tag{18}$$

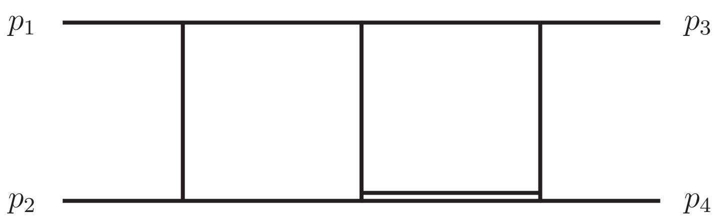

Figure 3: Drell-Yan with one internal massive line, the double line indicates a massive propagator.

The topology has a basis of 25 master integrals:

$$\begin{aligned}\vec{g}^{\,\mathrm{DYOM}}(\epsilon,s,t,m) \;=\; \big(&I_{\mathrm{DYOM}}(0,0,1,1,0,1,0,0,0), && I_{\mathrm{DYOM}}(1,0,1,1,0,1,0,0,0),\\ &I_{\mathrm{DYOMx124}}(1,0,0,1,0,0,1,0,0), && I_{\mathrm{DYOM}}(1,0,0,1,0,0,1,0,0),\\ &I_{\mathrm{DYOM}}(0,0,1,1,0,0,1,0,0), && I_{\mathrm{DYOM}}(1,0,1,1,0,0,1,0,0),\\ &I_{\mathrm{DYOM}}(1,0,1,0,1,0,1,0,0), && I_{\mathrm{DYOM}}(0,1,1,0,1,0,1,0,0),\\ &I_{\mathrm{DYOM}}(1,1,1,0,1,0,1,0,0), && I_{\mathrm{DYOM}}(1,1,0,1,1,0,1,0,0),\\ &I_{\mathrm{DYOM}}(1,0,1,1,1,0,1,0,0), && I_{\mathrm{DYOM}}(0,1,1,1,1,0,1,0,0),\\ &I_{\mathrm{DYOM}}(1,1,1,1,1,0,1,0,0), && I_{\mathrm{DYOM}}(0,0,1,0,0,1,1,0,0),\\ &I_{\mathrm{DYOM}}(-1,0,1,0,0,1,1,0,0), && I_{\mathrm{DYOM}}(0,1,0,1,0,1,1,0,0),\\ &I_{\mathrm{DYOM}}(0,1,1,1,0,1,1,0,0), && I_{\mathrm{DYOM}}(-1,1,1,1,0,1,1,0,0),\\ &I_{\mathrm{DYOM}}(0,1,1,0,1,1,1,0,0), && I_{\mathrm{DYOM}}(-1,1,1,0,1,1,1,0,0),\\ &I_{\mathrm{DYOM}}(0,1,0,1,1,1,1,0,0), && I_{\mathrm{DYOM}}(0,1,1,1,1,1,1,0,0),\\ &I_{\mathrm{DYOM}}(-1,1,1,1,1,1,1,0,0), && I_{\mathrm{DYOM}}(1,1,1,1,1,1,1,0,0),\\ &I_{\mathrm{DYOM}}(1,1,1,1,1,1,1,-1,0)\big)\,. \end{aligned} \tag{19}$$

The mass-dimension is factored out of the master integrals as follows

$$f_i(\epsilon,x,y) = (m)^{-\dim(g_i)} g_i(\epsilon,s,t,m), \tag{20}$$

with

$$x = \frac{s}{m^2}, \quad y = \frac{t}{m^2}. \tag{21}$$

4 Massless planar double box

This integral topology has first been computed in [7] and a treatment with the differential equations approach can be found in [8]. The topology is given by the following set of propagators

$$\begin{aligned} &P_1 = l_1^2, && P_4 = (l_2-p_3-p_4)^2, && P_7 = (l_1-l_2)^2,\\ &P_2 = (l_1-p_3)^2, && P_5 = (l_2-p_1)^2, && P_8 = (l_2-p_3)^2,\\ &P_3 = (l_1-p_3-p_4)^2, && P_6 = l_2^2, && P_9 = (l_1-p_1)^2. \end{aligned} \tag{22}$$

The momenta p_1 and p_2 are incoming and p_3 and p_4 are outgoing. The kinematics are given by

$$p_1^2 = 0, \quad p_2^2 = 0, \quad p_3^2 = 0, \quad p_4^2 = 0, \tag{23}$$

$$p_1 + p_2 = p_3 + p_4, \tag{24}$$

$$s = (p_1+p_2)^2, \quad t = (p_1-p_3)^2. \tag{25}$$

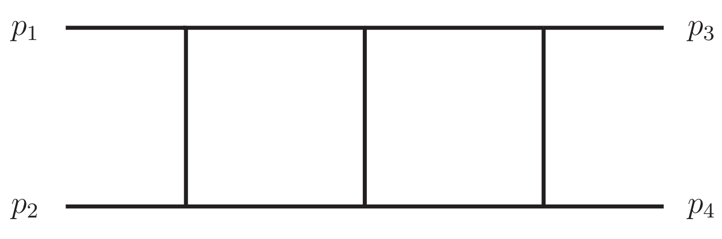

Figure 4: Massless planar double box.

The topology has a basis of 8 master integrals:

$$\begin{aligned}\vec{g}^{\,\mathrm{DBP}}(\epsilon,s,t) \;=\; \big(&I_{\mathrm{DBP}}(1,0,1,1,0,1,0,0,0), && I_{\mathrm{DBPx124}}(1,0,0,1,0,0,1,0,0),\\ &I_{\mathrm{DBP}}(1,0,0,1,0,0,1,0,0), && I_{\mathrm{DBP}}(1,0,1,0,1,0,1,0,0),\\ &I_{\mathrm{DBP}}(1,1,1,0,1,0,1,0,0), && I_{\mathrm{DBP}}(1,1,0,1,1,0,1,0,0),\\ &I_{\mathrm{DBP}}(1,1,1,1,1,1,1,-1,0), && I_{\mathrm{DBP}}(1,1,1,1,1,1,1,0,0)\big)\,. \end{aligned} \tag{26}$$

The mass-dimension is factored out of the master integrals as follows

$$f_i(\epsilon, x) = (t)^{-\dim(g_i)/2} g_i(\epsilon, s, t), \tag{27}$$

with

$$x = \frac{s}{t}. \tag{28}$$

5 Massless non-planar double box

This topology has first been computed in [9] and a treatment using differential equations can be found in [10]. The topology is given by the following set of propagators

$$\begin{array}{lll} P_1 = l_1^2, & P_4 = l_2^2, & P_7 = (l_1 - l_2 + p_3 - p_1)^2, \\ P_2 = (l_1 - p_4)^2, & P_5 = (l_2 - l_1 - p_3)^2, & P_8 = (l_1 + p_2)^2, \\ P_3 = (l_2 - p_2)^2, & P_6 = (l_1 + p_3)^2, & P_9 = (l_2 - p_3)^2. \end{array} \tag{29}$$

The momenta p_1 and p_2 are incoming and p_3 and p_4 are outgoing. The kinematics are given by

$$p_1^2 = 0, \quad p_2^2 = 0, \quad p_3^2 = 0, \quad p_4^2 = 0, \tag{30}$$

$$p_1 + p_2 = p_3 + p_4, \tag{31}$$

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2. \tag{32}$$

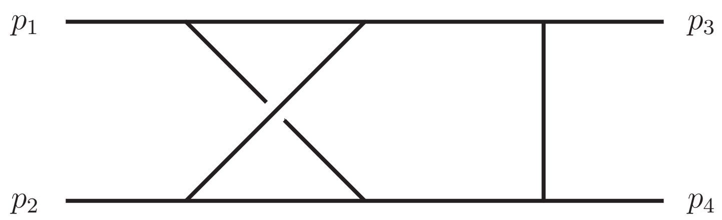

Figure 5: Massless non-planar double box

The topology has a basis of 12 master integrals:

$$\begin{array}{rcll} \vec{g}^{\,\mathrm{DBPNP}}(\epsilon, s, t) & = & \big(I_{\mathrm{DBPNPx124}}(1,0,1,0,1,0,0,0,0), & I_{\mathrm{DBPNPx12}}(1,0,1,0,1,0,0,0,0), \\ & & I_{\mathrm{DBNP}}(1,0,1,0,1,0,0,0,0), & I_{\mathrm{DBPNPx123}}(1,1,1,1,1,0,0,0,0), \\ & & I_{\mathrm{DBPNPx12}}(1,1,1,1,1,0,0,0,0), & I_{\mathrm{DBNP}}(1,1,1,1,1,0,0,0,0), \\ & & I_{\mathrm{DBNP}}(0,1,1,0,1,1,0,0,0), & I_{\mathrm{DBPNPx12}}(1,1,1,0,1,1,0,0,0), \\ & & I_{\mathrm{DBNP}}(1,1,1,0,1,1,0,0,0), & I_{\mathrm{DBNP}}(0,1,1,1,1,1,1,0,0), \\ & & I_{\mathrm{DBNP}}(1,1,1,1,1,1,1,0,0), & I_{\mathrm{DBNP}}(1,1,1,1,1,1,1,-1,0)\big). \end{array} \tag{33}$$

The mass-dimension is factored out of the master integrals as follows

$$f_i(\epsilon, x) = (t)^{-\dim(g_i)/2} g_i(\epsilon, s, t), \tag{34}$$

with

$$x = \frac{s}{t}. \tag{35}$$

6 Single top-quark topology 1

This topology has already been presented as an example in [11]. The topology is given by the following set of propagators

$$\begin{array}{lll} P_1 = l_2^2, & P_4 = (l_2+p_2)^2, & P_7 = (l_1+l_2-p_1+p_3)^2, \\ P_2 = l_1^2 - m_W^2, & P_5 = (l_1-p_4)^2, & P_8 = (l_1-p_2)^2, \\ P_3 = (l_1+p_3)^2, & P_6 = (l_2-p_1)^2, & P_9 = (l_2+p_3+p_1)^2. \end{array} \tag{36}$$

The momenta p_1 and p_2 are incoming and p_3 and p_4 are outgoing. The kinematics are given by

$$p_1^2 = 0, \quad p_2^2 = 0, \quad p_3^2 = 0, \quad p_4^2 = m_t^2, \tag{37}$$

$$p_1 + p_2 = p_3 + p_4, \tag{38}$$

$$s = (p_1+p_2)^2, \quad t = (p_2-p_3)^2. \tag{39}$$

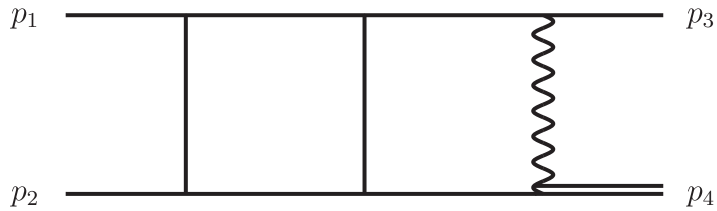

Figure 6: Single top-quark topology 1.

The topology has a basis of 31 master integrals:

$$\begin{aligned} \vec{g}^{\,\mathrm{t1}}(\epsilon, s, t, m_t^2, m_W^2) \;=\; \big(& I_{\mathrm{t1}}(0,1,0,1,0,1,0,0,0), && I_{\mathrm{t1}}(0,1,0,1,1,1,0,0,0), \\ & I_{\mathrm{t1}}(0,0,1,1,1,1,0,0,0), && I_{\mathrm{t1}}(0,1,1,1,1,1,0,0,0), \\ & I_{\mathrm{t1}}(1,1,0,0,0,0,1,0,0), && I_{\mathrm{t1}}(1,1,-1,0,0,0,1,0,0), \\ & I_{\mathrm{t1}}(0,1,0,1,0,0,1,0,0), && I_{\mathrm{t1}}(-1,1,0,1,0,0,1,0,0), \\ & I_{\mathrm{t1}}(0,0,1,1,0,0,1,0,0), && I_{\mathrm{t1}}(0,1,1,1,0,0,1,0,0), \\ & I_{\mathrm{t1}}(1,1,1,1,0,0,1,0,0), && I_{\mathrm{t1}}(1,1,1,1,-1,0,1,0,0), \\ & I_{\mathrm{t1}}(1,1,0,0,1,0,1,0,0), && I_{\mathrm{t1}}(1,0,1,0,1,0,1,0,0), \\ & I_{\mathrm{t1}}(1,1,1,0,1,0,1,0,0), && I_{\mathrm{t1}}(1,1,1,-1,1,0,1,0,0), \\ & I_{\mathrm{t1}}(0,1,0,0,0,1,1,0,0), && I_{\mathrm{t1}}(0,1,0,1,0,1,1,0,0), \\ & I_{\mathrm{t1}}(-1,1,0,1,0,1,1,0,0), && I_{\mathrm{t1}}(1,1,0,1,0,1,1,0,0), \\ & I_{\mathrm{t1}}(1,1,-1,1,0,1,1,0,0), && I_{\mathrm{t1}}(0,1,1,1,0,1,1,0,0), \\ & I_{\mathrm{t1}}(0,1,0,0,1,1,1,0,0), && I_{\mathrm{t1}}(-1,1,0,0,1,1,1,0,0), \\ & I_{\mathrm{t1}}(1,1,0,0,1,1,1,0,0), && I_{\mathrm{t1}}(1,1,-1,0,1,1,1,0,0), \\ & I_{\mathrm{t1}}(0,1,0,1,1,1,1,0,0), && I_{\mathrm{t1}}(1,1,0,1,1,1,1,0,0), \\ & I_{\mathrm{t1}}(1,1,1,1,1,1,1,0,0), && I_{\mathrm{t1}}(1,1,1,1,1,1,1,0,-1), \\ & I_{\mathrm{t1}}(1,1,1,1,1,1,1,-1,0)\big). \end{aligned} \tag{40}$$

The mass-dimension is factored out of the master integrals as follows

$$f_i(\epsilon, x, y, z) = (m_W)^{-\dim(g_i)} g_i(\epsilon, s, t, m_t, m_W), \tag{41}$$

with

$$x = \frac{s}{m_W^2}, \quad y = \frac{t}{m_W^2}, \quad z = \frac{m_t^2}{m_W^2}. \tag{42}$$

7 Single top-quark topology 2

This topology has already been presented as an example in [11]. The topology is given by the following set of propagators

$$\begin{array}{lll} P_1 = l_2^2, & P_4 = (l_2 - p_2)^2, & P_7 = (l_1 - l_2 - p_1 + p_3)^2, \\ P_2 = l_1^2 - m_W^2, & P_5 = (l_1 - p_4)^2, & P_8 = (l_1 + p_2)^2, \\ P_3 = (l_1 + p_3)^2, & P_6 = (l_2 - l_1 - p_3)^2, & P_9 = (l_2 - p_3)^2. \end{array} \tag{43}$$

The momenta p_1 and p_2 are incoming and p_3 and p_4 are outgoing. The kinematics are given by

$$p_1^2 = 0, \quad p_2^2 = 0, \quad p_3^2 = 0, \quad p_4^2 = m_t^2, \tag{44}$$

$$p_1 + p_2 = p_3 + p_4, \tag{45}$$

$$s = (p_1 + p_2)^2, \quad t = (p_2 - p_3)^2. \tag{46}$$

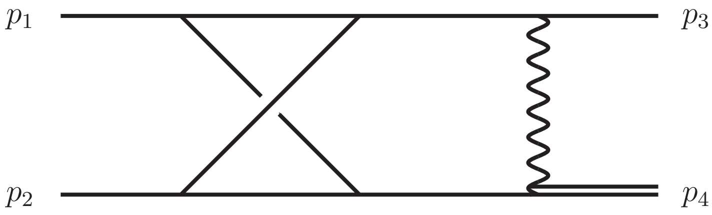

Figure 7: Single top-quark topology 2.

The topology has a basis of 35 master integrals:

$$\begin{aligned} \vec{g}^{\,\mathrm{t2}}(\epsilon, s, t, m_t^2, m_W^2) = \big(& I_{\mathrm{t2}}(1,1,0,0,0,1,0,0,0), && I_{\mathrm{t2x12}}(-1,1,0,1,0,1,0,0,0), \\ & I_{\mathrm{t2x12}}(0,1,0,1,0,1,0,0,0), && I_{\mathrm{t2}}(-1,1,0,1,0,1,0,0,0), \\ & I_{\mathrm{t2}}(0,1,0,1,0,1,0,0,0), && I_{\mathrm{t2}}(1,0,0,0,1,1,0,0,0), \\ & I_{\mathrm{t2}}(1,1,-1,0,1,1,0,0,0), && I_{\mathrm{t2}}(1,1,0,0,1,1,0,0,0), \\ & I_{\mathrm{t2x12}}(0,1,0,1,1,1,0,0,0), && I_{\mathrm{t2}}(0,1,0,1,1,1,0,0,0), \\ & I_{\mathrm{t2x12}}(1,1,-1,1,1,1,0,0,0), && I_{\mathrm{t2x12}}(1,1,0,1,1,1,0,0,0), \\ & I_{\mathrm{t2}}(1,1,-1,1,1,1,0,0,0), && I_{\mathrm{t2}}(1,1,0,1,1,1,0,0,0), \\ & I_{\mathrm{t2}}(0,0,1,1,1,1,0,0,0), && I_{\mathrm{t2x12}}(-1,1,1,1,1,1,0,0,0), \\ & I_{\mathrm{t2x12}}(0,1,1,1,1,1,0,0,0), && I_{\mathrm{t2}}(-1,1,1,1,1,1,0,0,0), \\ & I_{\mathrm{t2}}(0,1,1,1,1,1,0,0,0), && I_{\mathrm{t2}}(-1,1,0,1,0,0,1,0,0), \\ & I_{\mathrm{t2}}(0,1,-1,1,0,0,1,0,0), && I_{\mathrm{t2}}(0,1,1,1,0,0,1,0,0), \\ & I_{\mathrm{t2x12}}(1,1,1,1,-1,0,1,0,0), && I_{\mathrm{t2x12}}(1,1,1,1,0,0,1,0,0), \\ & I_{\mathrm{t2}}(1,1,1,1,-1,0,1,0,0), && I_{\mathrm{t2}}(1,1,1,1,0,0,1,0,0), \\ & I_{\mathrm{t2}}(1,1,-1,1,0,1,1,0,0), && I_{\mathrm{t2}}(1,1,0,1,0,1,1,0,0), \\ & I_{\mathrm{t2}}(1,1,1,1,0,1,1,0,0), && I_{\mathrm{t2}}(1,1,-1,1,1,1,1,0,0), \\ & I_{\mathrm{t2}}(1,1,0,1,1,1,1,0,0), && I_{\mathrm{t2}}(1,0,1,1,1,1,1,0,0), \\ & I_{\mathrm{t2}}(1,1,1,1,1,1,1,-2,0), && I_{\mathrm{t2}}(1,1,1,1,1,1,1,-1,0), \\ & I_{\mathrm{t2}}(1,1,1,1,1,1,1,0,0)\big). \end{aligned} \tag{47}$$

The mass-dimension is factored out of the master integrals as follows

$$f_i(\epsilon, x, y, z) = (m_W)^{-\dim(g_i)} g_i(\epsilon, s, t, m_t, m_W), \tag{48}$$

with

$$x = \frac{s}{m_W^2}, \quad y = \frac{t}{m_W^2}, \quad z = \frac{m_t^2}{m_W^2}. \tag{49}$$

8 Vector boson pair production topology 1

This topology has been considered in [12–17]. It has also already been presented as an example in [11]. The topology is given by the following set of propagators

$$\begin{array}{lll} P_1 = l_1^2, & P_4 = (l_2 - p_3 - p_4)^2, & P_7 = (l_2 - p_1)^2, \\ P_2 = (l_1 - p_3 - p_4)^2, & P_5 = (l_1 - p_3)^2, & P_8 = (l_2 - p_3)^2, \\ P_3 = l_2^2, & P_6 = (l_1 - l_2)^2, & P_9 = (l_1 - p_1)^2. \end{array} \tag{50}$$

The momenta p_1 and p_2 are incoming and p_3 and p_4 are outgoing. The kinematics are given by

$$p_1^2 = 0, \quad p_2^2 = 0, \quad p_3^2 = m_3^2, \quad p_4^2 = m_4^2, \tag{51}$$

$$p_1 + p_2 = p_3 + p_4, \tag{52}$$

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2. \tag{53}$$

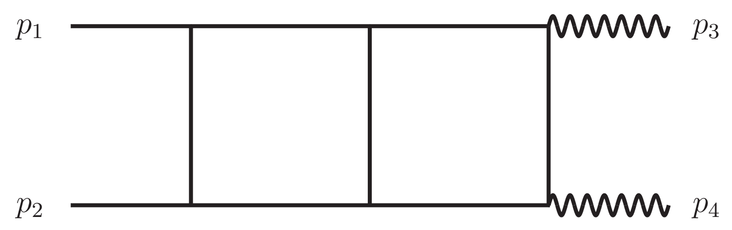

Figure 8: Vector boson pair production topology 1.

The topology has a basis of 31 master integrals:

$$\begin{aligned} \vec{g}^{\,\mathrm{t1}}(\epsilon, s, t, m_3, m_4) \;=\; & \big(I_{\mathrm{t1}}(1,1,1,1,0,0,0,0,0), \quad I_{\mathrm{t1}}(1,0,1,1,1,0,0,0,0), \\ & I_{\mathrm{t1}}(0,1,1,1,1,0,0,0,0), \quad I_{\mathrm{t1}}(1,1,1,1,1,0,0,0,0), \\ & I_{\mathrm{t1}}(0,1,1,0,0,1,0,0,0), \quad I_{\mathrm{t1}}(0,0,1,0,1,1,0,0,0), \\ & I_{\mathrm{t1}}(-1,1,1,0,1,1,0,0,0), \quad I_{\mathrm{t1}}(0,1,1,0,1,1,0,0,0), \\ & I_{\mathrm{t1}}(0,0,0,1,1,1,0,0,0), \quad I_{\mathrm{t1}}(1,-1,0,1,1,1,0,0,0), \\ & I_{\mathrm{t1}}(1,0,0,1,1,1,0,0,0), \quad I_{\mathrm{t1}}(-1,0,1,1,1,1,0,0,0), \\ & I_{\mathrm{t1}}(0,0,1,1,1,1,0,0,0), \quad I_{\mathrm{t1}}(1,0,1,1,1,1,0,0,0), \\ & I_{\mathrm{t1}}(0,1,1,1,1,1,0,0,0), \quad I_{\mathrm{t1}}(1,1,0,0,0,1,1,0,0), \\ & I_{\mathrm{t1}}(0,0,0,0,1,1,1,0,0), \quad I_{\mathrm{t1}}(1,0,0,0,1,1,1,0,0), \\ & I_{\mathrm{t1}}(0,1,0,0,1,1,1,0,0), \quad I_{\mathrm{t1}}(1,1,-1,0,1,1,1,0,0), \\ & I_{\mathrm{t1}}(1,1,0,0,1,1,1,0,0), \quad I_{\mathrm{t1}}(-1,1,1,0,1,1,1,0,0), \\ & I_{\mathrm{t1}}(0,1,1,0,1,1,1,0,0), \quad I_{\mathrm{t1}}(1,-1,0,1,1,1,1,0,0), \\ & I_{\mathrm{t1}}(1,0,0,1,1,1,1,0,0), \quad I_{\mathrm{t1}}(0,0,1,1,1,1,1,0,0), \\ & I_{\mathrm{t1}}(1,0,1,1,1,1,1,0,0), \quad I_{\mathrm{t1}}(0,1,1,1,1,1,1,0,0), \\ & I_{\mathrm{t1}}(1,1,1,1,1,1,1,-1,0), \quad I_{\mathrm{t1}}(1,1,1,1,1,1,1,0,-1), \\ & I_{\mathrm{t1}}(1,1,1,1,1,1,1,0,0)\big). \end{aligned} \tag{54}$$

The mass-dimension is factored out of the master integrals as follows

$$f_i(\epsilon, x, y, z) = (m_W)^{-\dim(g_i)} g_i(\epsilon, s, t, m_t, m_W), \tag{55}$$

with

$$x = \frac{s}{m_W^2}, \quad y = \frac{t}{m_W^2}, \quad z = \frac{m_t^2}{m_W^2}. \tag{56}$$

9 Vector boson pair production topology 2

This topology has been considered in [12–17]. It has also already been presented as an example in [11]. The topology is given by the following set of propagators

$$\begin{array}{lll} P_1 = l_1^2, & P_4 = (l_2 + p_1 - p_3)^2, & P_7 = (l_2 + p_4)^2, \\ P_2 = (l_1 + p_1 - p_3)^2, & P_5 = (l_1 - p_3)^2, & P_8 = (l_2 - p_3)^2, \\ P_3 = l_2^2, & P_6 = (l_1 - l_2)^2, & P_9 = (l_1 + p_4)^2. \end{array} \tag{57}$$

The momenta p_1 and p_2 are incoming and p_3 and p_4 are outgoing. The kinematics are given by

$$p_1^2 = 0, \quad p_2^2 = 0, \quad p_3^2 = m_3^2, \quad p_4^2 = m_4^2, \tag{58}$$

$$p_1 + p_2 = p_3 + p_4, \tag{59}$$

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2. \tag{60}$$

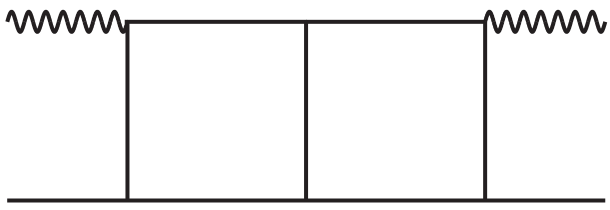

Figure 9: Vector boson pair production topology 2.

The topology has a basis of 29 master integrals:

$$\begin{array}{rcll} \vec{g}^{\,\mathrm{t}2}(\epsilon, s, t, m_3, m_4) & = & \big(I_{\mathrm{t}2}(1,1,1,1,0,0,0,0,0), & I_{\mathrm{t}2}(1,0,1,1,1,0,0,0,0), \\ & & I_{\mathrm{t}2}(0,1,1,0,0,1,0,0,0), & I_{\mathrm{t}2}(0,0,1,0,1,1,0,0,0), \\ & & I_{\mathrm{t}2}(1,0,0,1,1,1,0,0,0), & I_{\mathrm{t}2}(0,0,1,1,1,1,0,0,0), \\ & & I_{\mathrm{t}2}(1,0,1,1,1,1,0,0,0), & I_{\mathrm{t}2}(1,1,1,0,0,0,1,0,0), \\ & & I_{\mathrm{t}2}(1,0,1,0,1,0,1,0,0), & I_{\mathrm{t}2}(1,0,0,0,0,1,1,0,0), \\ & & I_{\mathrm{t}2}(1,1,0,0,0,1,1,0,0), & I_{\mathrm{t}2}(0,1,1,0,0,1,1,0,0), \\ & & I_{\mathrm{t}2}(1,1,1,0,0,1,1,0,0), & I_{\mathrm{t}2}(0,0,0,0,1,1,1,0,0), \\ & & I_{\mathrm{t}2}(1,-1,0,0,1,1,1,0,0), & I_{\mathrm{t}2}(1,0,0,0,1,1,1,0,0), \\ & & I_{\mathrm{t}2}(1,1,0,0,1,1,1,0,0), & I_{\mathrm{t}2}(-1,0,1,0,1,1,1,0,0), \\ & & I_{\mathrm{t}2}(0,0,1,0,1,1,1,0,0), & I_{\mathrm{t}2}(1,0,1,0,1,1,1,0,0), \\ & & I_{\mathrm{t}2}(-1,1,1,0,1,1,1,0,0), & I_{\mathrm{t}2}(0,1,1,0,1,1,1,0,0), \\ & & I_{\mathrm{t}2}(1,1,1,0,1,1,1,0,0), & I_{\mathrm{t}2}(1,-1,0,1,1,1,1,0,0), \\ & & I_{\mathrm{t}2}(1,0,0,1,1,1,1,0,0), & I_{\mathrm{t}2}(0,0,1,1,1,1,1,0,0), \\ & & I_{\mathrm{t}2}(1,0,1,1,1,1,1,0,0), & I_{\mathrm{t}2}(1,1,1,1,1,1,1,-1,0), \\ & & I_{\mathrm{t}2}(1,1,1,1,1,1,1,0,0)\big). & \end{array} \tag{61}$$

The mass-dimension is factored out of the master integrals as follows

$$f_i(\epsilon, x, y, z) = (m_W)^{-\dim(g_i)} g_i(\epsilon, s, t, m_t, m_W), \tag{62}$$

with

$$x = \frac{s}{m_W^2}, \quad y = \frac{t}{m_W^2}, \quad z = \frac{m_t^2}{m_W^2}. \tag{63}$$

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