Low-energy properties of hadrons in the relativistic quark model

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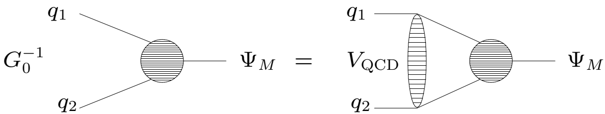
Outline

- Relativistic quark model
- Spectroscopy of mesons, tetraquarks and baryons
- Electroweak properties of hadrons
- Conclusions

Relativistic quark model (RQM)



Schrödinger-like quasipotential equation: (Logunov, Tavkhelidze)

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q})$$

$\mathbf{p} = (\mathbf{p}_2 - \mathbf{p}_1)/2$ is relative momentum of quarks
M is bound state mass ($M = E_1 + E_2$)
μ_R is relativistic reduced mass:

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$b^2(M)$ is relative momentum squared in center-of-mass frame on mass shell:

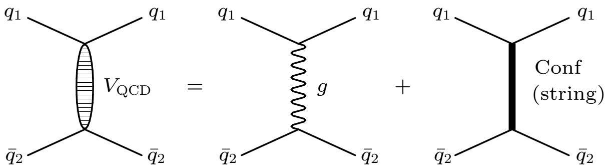
$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

$E_{1,2}$ are energies in center-of-mass frame:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

Quark interaction potential in hadrons

$q\bar{q}$ interaction in meson



consists of

- one-gluon exchange potential $V^{\rm Coul}$ with Lorentz-vector structure, which dominates at small distances in QCD;
- long-range linearly rising scalar confining potential $V^S_{\rm conf}$ and
- long-range linearly rising vector confining potential $V^V_{\rm conf}$, which dominate at large distances.

It is assumed that

- quarks have long-range effective anomalous chromomagnetic moment κ.
- quarks have fixed constituent masses

Projection on positive-energy states leads to quasipotential:

$$V(\mathbf{p},\mathbf{q};M)=\bar{u}_1(p)\bar{u}_2(-p)\left\{\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^{\mu}\gamma_2^{\nu}+V^V_{\rm conf}(\mathbf{k})\Gamma_1^{\mu}\Gamma_{2;\mu}+V^S_{\rm conf}(\mathbf{k})\right\}u_1(q)u_2(-q)$$

$\mathbf{k}=\mathbf{p}-\mathbf{q}$
$D_{\mu\nu}(\mathbf{k})$ is (perturbative) gluon potential
$\Gamma_{\mu}(\mathbf{k})$ is effective vertex of long-range vector interaction containing Pauli term:

$$\Gamma_{\mu}(\mathbf{k})=\gamma_{\mu}+\frac{i\kappa}{2m}\sigma_{\mu\nu}k^{\nu},$$

- κ is nonperturbative anomalous chromomagnetic moment of quark,

$$u^{\lambda}(p)=\sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\begin{pmatrix}1\\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m}\end{pmatrix}\chi^{\lambda},\qquad \epsilon(p)=\sqrt{\mathbf{p}^2+m^2}.$$

- Quasipotential $V(\mathbf{p},\mathbf{q};M)$ is strongly nonlocal in coordinate space
- Lorentz-structure of confining potential is mixture of vector $V^V_{\rm conf}$ and scalar $V^S_{\rm conf}$ interactions

In nonrelativistic limit vector and scalar potentials in coordinate space

$$V^V_{\rm conf}(r) = (1-\varepsilon)(Ar+B),$$
$$V^S_{\rm conf}(r) = \varepsilon(Ar+B),$$

- ε is mixing coefficient

$$V_{\rm conf}(r) = V^S_{\rm conf}(r) + V^V_{\rm conf}(r) = Ar + B,$$

Total $q\bar{q}$ potential

$$V_{q\bar{q}}(r) = V^{\rm Coul}(r) + V_{\rm conf}(r) = \frac{4}{3}\frac{\alpha_s}{r} + Ar + B$$

Running coupling constant $\alpha_s \equiv \alpha_s(\mu^2)$:
a) heavy hadrons

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)},$$

b) light hadrons (with freezing)

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln \dfrac{\mu^2 + M_0^2}{\Lambda^2}}, \qquad \beta_0 = 11 - \frac{2}{3}n_f,$$

$\mu = 2m_1 m_2/(m_1 + m_2)$, $\quad M_0 = 2.24\sqrt{A} \approx 0.95$ GeV

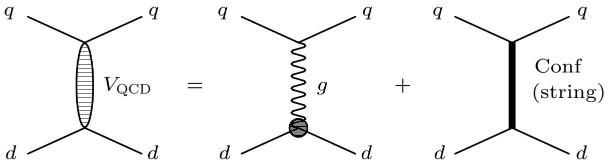
qq interaction in diquark

Potential of quark-quark interaction

$$V_{qq} = V_{q\bar{q}}/2,$$

$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\frac{1}{2}\left[\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + V^S_{\rm conf}(\mathbf{k})\right] u_1(q)u_2(-q).$$

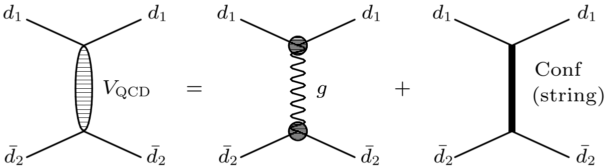
Quark-diquark interaction in baryon



$$V(\mathbf{p},\mathbf{q};M) = \frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d(p)E_d(q)}}\bar{u}_q(p)\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma^\nu u_q(q)$$
$$+\psi^*_d(P)\bar{u}_q(p)\left[J_{d;\mu}\Gamma_q^\mu(\mathbf{k})V^V_{\rm conf}(\mathbf{k}) + V^S_{\rm conf}(\mathbf{k})\right] u_q(q)\psi_d(Q)$$

$\langle d(P)|J_\mu|d(Q)\rangle$ is vertex of diquark-gluon interaction

Diquark-antidiquark interaction in tetraquark



$$
\begin{aligned}
V(\mathbf{p},\mathbf{q};M) &= \frac{\langle d_1(P_1)|J_\mu|d_1(Q_1)\rangle}{2\sqrt{E_{d_1}(p)E_{d_1}(q)}}\,\frac{4}{3}\alpha_s D^{\mu\nu}(\mathbf{k})\,\frac{\langle d_2(P_2)|J_\nu|d_2(Q_2)\rangle}{2\sqrt{E_{d_2}(p)E_{d_2}(q)}} \\
&\quad+\psi^*_{d_1}(P_1)\psi^*_{d_2}(P_2)\left[J_{d_1;\mu}J^\mu_{d_2}V^V_{\rm conf}(\mathbf{k})+V^S_{\rm conf}(\mathbf{k})\right]\psi_{d_1}(Q_1)\psi_{d_2}(Q_2).
\end{aligned}
$$

Free parameters of the model:

- constituent quark masses: $m_{u,d} = 0.33$ GeV, $m_s = 0.50$ GeV, $m_c = 1.55$ GeV, $m_b = 4.88$ GeV;
- slope of linear confining potential $A = 0.18$ GeV^2;
- constant B in confining potential $B = -0.30$ GeV;
- scale Λ of the running constant α_s:
  $\Lambda = 0.169$ GeV for heavy hadrons
  $\Lambda = 0.413$ GeV for light hadrons;
- mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$;
- long-range anomalous chromomagnetic moment of quark $\kappa = -1$;

All free parameters (10) were fixed from analysis of meson mass spectra and radiative decays.

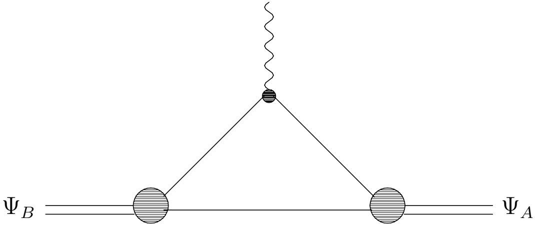
• Values of parameters ε and κ are confirmed by comparison with predictions of heavy quark effective theory (HQET), instanton vacuum and flux tube models.

Matrix element of current between bound states

Matrix element of current between bound states in quasipotential approach

$$\langle A|J_\mu(0)|B\rangle = \int \frac{d^3p\,d^3q}{(2\pi)^6}\bar{\Psi}_{A\,\mathbf{P}}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_{B\,\mathbf{Q}}(\mathbf{q}).$$

In initial approximation



- Relativistic transformation of wave function from center-of-mass to moving frame:

a) bound system of quarks

$$\Psi_{\mathbf{P}}(\mathbf{p}) = D_1^{1/2}(R_{L_{\mathbf{P}}}^W)D_2^{1/2}(R_{L_{\mathbf{P}}}^W)\Psi_{\mathbf{0}}(\mathbf{p}),$$

$\Psi_{\mathbf{0}}(\mathbf{p}) \equiv \Psi_M(\mathbf{p})$ is wave function in center-of-mass frame; R^W is Wigner rotation, $L_{\mathbf{P}}$ is Lorentz transformation from center-of-mass to moving frame, $D^{1/2}(R)$ are rotation matrices

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D_{1,2}^{1/2}(R_{L_{\mathbf{P}}}^W) = S^{-1}(\mathbf{p}_{1,2})S(\mathbf{P})S(\mathbf{p}),$$

$$S(\mathbf{p}) = \sqrt{\frac{\epsilon(p)+m}{2m}}\left(1 + \frac{\boldsymbol{\alpha}\mathbf{p}}{\epsilon(p)+m}\right).$$

b) quark-diquark bound system

$$\Psi_{B_Q\,\mathbf{P}}(\mathbf{p}) = D_Q^{1/2}(R_{L_{\mathbf{P}}}^W)D_d^{\mathcal{I}}(R_{L_{\mathbf{P}}}^W)\Psi_{B_Q\,\mathbf{0}}(\mathbf{p}), \qquad \mathcal{I} = 0, 1,$$

$$D^{\mathcal{I}}(R^W) = \begin{cases} 1 & \text{for scalar diquark } \mathcal{I} = 0 \\ R^W & \text{for axial vector diquark } \mathcal{I} = 1 \end{cases}.$$

Hadron spectroscopy

- for heavy (b, c) quarks expansion in v/c or $1/m_Q$ can be applied

- light (u, d, s) quarks are highly relativistic ($v/c \sim 0.7 \div 0.8$) expansion in $1/m_q$ (or v/c) is not applicable and fully relativistic treatment is necessary. This leads to the quasipotential which is nonlocal in coordinate space

- substitution of light quark energy $\epsilon_q(p)$ by energy E_q

$$\epsilon_q(p) = \sqrt{m_q^2 + \mathbf{p}^2} \to E_q = \frac{M^2 - m_Q^2 + m_q^2}{2M}$$

makes quasipotential local, but nonlinearly dependent on hadron mass M

- effective methods of numerical solution of quasipotential equation were developed

- freezing of running constant α_s for light hadrons is taken into account

- baryons are considered in quark-diquark approximation (successive solution of 2 two-body problems)

- internal structure of diquark is taken into account with the help of form factor of diquark-gluon interaction

Charmonium, bottomonium and B_c meson spectroscopy

Table: Charmonium mass spectrum (in GeV).

State $n\,^{2S+1}L_J$	Meson	Theory	Experiment PDG
$1\,^1S_0$	η_c	2.979	2.9804
$1\,^3S_1$	J/Ψ	3.096	3.096916
$1\,^3P_0$	χ_{c0}	3.424	3.41476
$1\,^3P_1$	χ_{c1}	3.510	3.51066
$1\,^3P_2$	χ_{c2}	3.556	3.55620
$1\,^1P_1$	h_c	3.526	3.52567
$2\,^1S_0$	η_c'	3.633	3.638(4)
$2\,^3S_1$	Ψ'	3.686	3.686093
$1\,^3D_1$		3.798	3.7711(24)
$1\,^3D_2$		3.813	
$1\,^3D_3$		3.815	
$1\,^1D_2$		3.811	
$2\,^3P_0$	χ_{c0}'	3.854	
$2\,^3P_1$	χ_{c1}'	3.929	
$2\,^3P_2$	χ_{c2}'	3.972	3.929(5)?
$2\,^1P_1$	h_c'	3.945	
$3\,^1S_0$	η_c''	3.991	3.943(12)?
$3\,^3S_1$	Ψ''	4.088	4.039(1)

$M^{\rm exp}_{\rm cog}(1^3P_J) = 3525.36$ MeV; $M^{\rm exp}_{h_c}(1^1P_1) - M^{\rm exp}_{\rm cog}(1^3P_J) = 0.31$ MeV

$M^{\rm th}_{\rm cog}(1^3P_J) = 3525$ MeV

Table: Bottomonium mass spectrum (in GeV).

State $n^{2S+1}L_J$	Meson	Theory	Experiment PDG
$1\,^1S_0$	η_b	9.400	9.3929(60)*
$1\,^3S_1$	Υ	9.460	9.46030
$1\,^3P_0$	χ_{b0}	9.863	9.85944
$1\,^3P_1$	χ_{b1}	9.892	9.89278
$1\,^3P_2$	χ_{b2}	9.913	9.91221
$1\,^1P_1$	h_b	9.901	
$2\,^1S_0$	η_b'	9.993	
$2\,^3S_1$	Υ'	10.023	10.02326
$1\,^3D_1$		10.153	
$1\,^3D_2$		10.158	10.162
$1\,^3D_3$		10.162	
$1\,^1D_2$		10.158	
$2\,^3P_0$	χ_{b0}'	10.234	10.2325
$2\,^3P_1$	χ_{b1}'	10.255	10.25546
$2\,^3P_2$	χ_{b2}'	10.268	10.26865
$2\,^1P_1$	h_b'	10.261	
$3\,^1S_0$	η_b''	10.328	
$3\,^3S_1$	Υ''	10.355	10.3552

* BaBar 2009

Table: B_c meson mass spectrum (in GeV).

State $n\,^{2S+1}L_J$	RQM	Eichten et al.	Gershtein et al.	Fulcher	Nusinov et al.
$1\,^1S_0$	6.270	6.264	6.253	6.286	≥ 6.2196
$1\,^3S_1$	6.332	6.337	6.317	6.341	≥ 6.2786
$1\,^3P_0$	6.699	6.700	6.683	6.701	≥ 6.6386
$1P1$	6.734	6.730	6.717	6.737	≥ 6.7012
$1P1'$	6.749	6.736	6.729	6.760	≥ 6.7012
$1\,^3P_2$	6.762	6.747	6.743	6.772	≥ 6.7347
$2\,^1S_0$	6.835	6.856	6.867	6.882	
$2\,^3S_1$	6.881	6.899	6.902	6.914	
$1\,^3D_1$	7.072	7.012	7.008	7.019	
$1D2$	7.077	7.009	7.001	7.028	
$1D2'$	7.079	7.012	7.016	7.028	
$1\,^3D_3$	7.081	7.005	7.007	7.032	
$2\,^3P_0$	7.091	7.108	7.088		
$2P1$	7.126	7.135	7.113		
$2P1'$	7.145	7.142	7.124		
$2\,^3P_2$	7.156	7.153	7.134		
$3\,^1S_0$	7.193	7.244			
$3\,^3S_1$	7.235	7.280			

CDF 2008 value (from $B_c \to J/\Psi\pi$):

$$M_{B_c} = 6275.6 \pm 2.9 \pm 2.5 \text{ MeV.}$$

Heavy-light meson spectroscopy

Table: D meson mass spectra (in MeV).

$(n\,^{2S+1}L_J)$	Meson	Theory	Experiment	Meson	Theory	Experiment
$1\,^1S_0$	D	1871	1869.62(20)	D_s	1969	1968.49(30)
$1\,^3S_1$	D^*	2010	2010.27(17)	D_s^*	2111	2112.3(5)
$1\,^3P_2$	D_2^*	2460	2460.1(3.5)	D_{s2}^*	2571	2572.8(9)
$1P_1$	D_1	2426	2423.4(31)	D_{s1}	2536	2535.35(80)
$1P_1$	D_1	2469	2427(40)	D_{s1}	2574	2459.6(6)
$1\,^3P_0$	D_0^*	2406	2403(40)	D_{s0}^*	2509	2317.8(6)
$2\,^1S_0$	D'	2581		D_s'	2698	
$2\,^3S_1$	$D^{*'}$	2632	2637(9)?	$D_s^{*'}$	2741	2690(7)?

Table: B meson mass spectra (in MeV).

$(n\,^{2S+1}L_J)$	Meson	Theory	Experiment	Meson	Theory	Experiment
$1\,^1S_0$	B	5280	5279.5(3)	B_s	5372	5366.3(6)
$1\,^3S_1$	B^*	5326	5325.1(5)	B_s^*	5413	5415.4(1.4)
$1\,^3P_2$	B_2^*	5741	5743(5)	B_{s2}^*	5842	5839.7(6)
$1P_1$	B_1	5723	5723.4(2.0)	B_{s1}	5831	5829.4(7)
$1P_1$	B_1	5774		B_{s1}	5865	5853(15)
$1\,^3P_0$	B_0^*	5749		B_{s0}^*	5833	
$2\,^1S_0$	B'	5890		B_s'	5976	
$2\,^3S_1$	$B^{*'}$	5908		$B_s^{*'}$	5992	

Heavy-light meson Regge trajectories

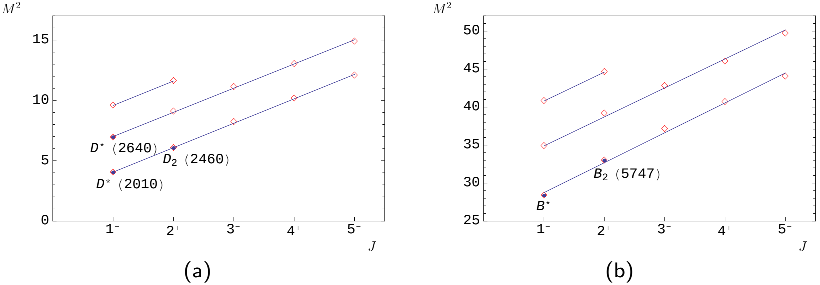


Figure: Parent and daughter (J, M^2) Regge trajectories for D (a) and B (b) heavy-light mesons with natural parity. Diamonds are predicted masses. Experimental data are given by dots with error bars. M^2 is in GeV^2.

Light meson spectroscopy

Table: Masses of excited light ($q = u, d$) unflavored mesons (in MeV).

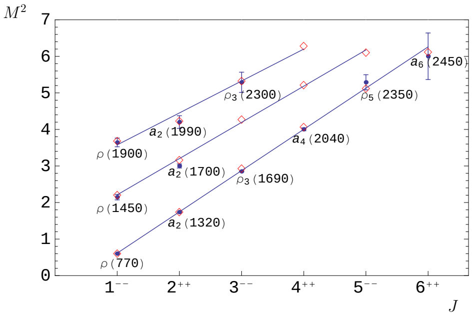
$n^{2S+1}L_J$	J^{PC}	Theory	Experiment				Theory	Experiment	
		$q\bar{q}$	$I = 1$	mass	$I = 0$	mass	$s\bar{s}$	$I = 0$	mass
1^1S_0	0^{-+}	154	π	139.57			743		
1^3S_1	1^{--}	776	ρ	775.49(34)	ω	782.65(12)	1038	φ	1019.455(20)
1^3P_0	0^{++}	1176	a_0	1474(19)	f_0	1200-1500	1420	f_0	1505(6)
1^3P_1	1^{++}	1254	a_1	1230(40)	f_1	1281.8(6)	1464	f_1	1426.4(9)
1^3P_2	2^{++}	1317	a_2	1318.3(6)	f_2	1275.1(12)	1529	f_2'	1525(5)
1^1P_1	1^{+-}	1258	b_1	1229.5(32)	h_1	1170(20)	1485	h_1	1386(19)
2^1S_0	0^{-+}	1292	π	1300(100)	η	1294(4)	1536	η	1476(4)
2^3S_1	1^{--}	1486	ρ	1465(25)	ω	1400-1450	1698	φ	1680(20)
1^3D_1	1^{--}	1557	ρ	1570(70)	ω	1670(30)	1845		
1^3D_2	2^{--}	1661					1908		
1^3D_3	3^{--}	1714	ρ_3	1688.8(21)	ω_3	1667(4)	1950	φ_3	1854(7)
1^1D_2	2^{-+}	1643	π_2	1672.4(32)	η_2	1617(5)	1909	η_2	1842(8)
2^3P_0	0^{++}	1679			f_0	1724(7)	1969		
2^3P_1	1^{++}	1742	a_1	1647(22)			2016	f_1	1971(15)
2^3P_2	2^{++}	1779	a_2	1732(16)	f_2	1755(10)	2030	f_2	2010(70)
2^1P_1	1^{+-}	1721					2024		
3^1S_0	0^{-+}	1788	π	1816(14)	η	1756(9)	2085	η	2103(50)
3^3S_1	1^{--}	1921	ρ	1909(31)	ω	1960(25)	2119	φ	2175(15)
1^3F_2	2^{++}	1797			f_2	1815(12)	2143	f_2	2156(11)
1^3F_3	3^{++}	1910	a_3	1874(105)			2215	f_3	2334(25)
1^3F_4	4^{++}	2018	a_4	2001(10)	f_4	2018(11)	2286		

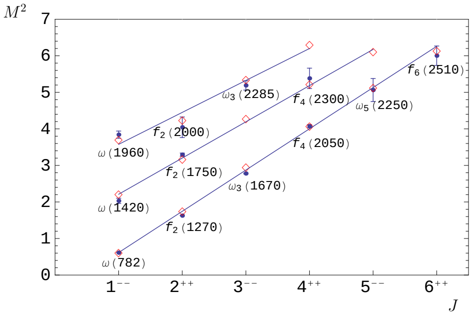
$M_\eta^{th} = 573$ MeV, $M_\eta^{exp} = 548$ MeV; $M_{\eta'}^{th} = 989$ MeV, $M_{\eta'}^{exp} = 958$ MeV

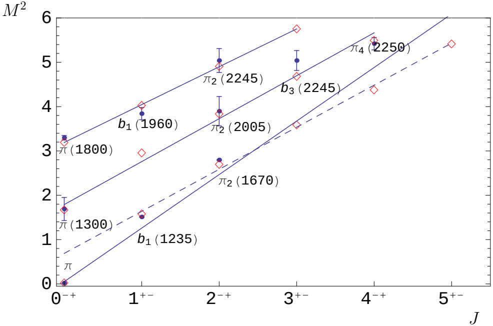
Table: Masses of strange mesons (in MeV).

$n^{2S+1}L_J$	J^P	Theory	Experiment	
		$q\bar{s}$		
1^1S_0	0^-	482	K	493.677(16)
1^3S_1	1^-	897	K^*	891.66(26)
1^3P_0	0^+	1362	K_0	1425(50)
1^3P_2	2^+	1424	K_2^*	1425.6(15)
$1P_1$	1^+	1412	K_1	1403(7)
$1P_1$	1^+	1294	K_1	1272(7)
2^1S_0	0^-	1538		
2^3S_1	1^-	1675	K^*	
1^3D_1	1^-	1699	K^*	1717(27)
1^3D_3	3^-	1789	K_3^*	1776(7)
$1D_2$	2^-	1824	K_2	1816(13)
$1D_2$	2^-	1709	K_2	1773(8)
2^3P_0	0^+	1791		
2^3P_2	2^+	1896		
$2P_1$	1^+	1893		
$2P_1$	1^+	1757	K_1	1650(50)

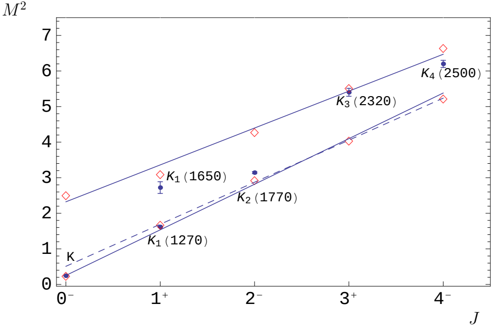
Regge trajectories in (J, M^2) plane (M^2 in GeV^2)







M^2
6
5
4
3
2
1
0
$\pi_4(2250)$
$\pi_2(2245)$
$b_3(2245)$
$b_1(1960)$
$\pi_2(2005)$
$\pi(1800)$
$\pi_2(1670)$
$\pi(1300)$
$b_1(1235)$
π
0^{-+}
1^{+-}
2^{-+}
3^{+-}
4^{-+}
5^{+-}
J



M^2
7
6
5
4
3
2
1
0
$K_4(2500)$
$K_3(2320)$
$K_1(1650)$
$K_2(1770)$
$K_1(1270)$
K
0^-
1^+
2^-
3^+
4^-
J

Heavy baryon spectroscopy

Table: Masses of ground state heavy baryons (in MeV).

Baryon	$I(J^P)$	Theory					Experiment
		RQM (2005)	Roncaglia et al	Karliner et al	Jenkins	Lewis Woloshyn	PDG
Λ_c	$0(\frac{1}{2}^+)$	2297	2285				2286.46(14)
Σ_c	$1(\frac{1}{2}^+)$	2439	2453				2453.76(18)
Σ_c^*	$1(\frac{3}{2}^+)$	2518	2520				2518.0(5)
Ξ_c	$\frac{1}{2}(\frac{1}{2}^+)$	2481	2468				2471.0(4)
Ξ_c'	$\frac{1}{2}(\frac{1}{2}^+)$	2578	2580		2580.8(2.1)		2578.0(2.9)
Ξ_c^*	$\frac{1}{2}(\frac{3}{2}^+)$	2654	2650				2646.1(1.2)
Ω_c	$0(\frac{1}{2}^+)$	2698	2710				2697.5(2.6)
Ω_c^*	$0(\frac{3}{2}^+)$	2768	2770		2760.5(4.9)		2768.3(3.0)†
Λ_b	$0(\frac{1}{2}^+)$	5622	5620			$5628(^{23}_{50})$	5620.2(1.6)
Σ_b	$1(\frac{1}{2}^+)$	5805	5820	5814	5824.2(9.0)	$5793(^{17}_{21})$	5807.5(2.5)‡
Σ_b^*	$1(\frac{3}{2}^+)$	5834	5850	5836	5840.0(8.8)	$5814(^{26}_{27})$	5829.0(2.3)‡
Ξ_b	$\frac{1}{2}(\frac{1}{2}^+)$	5812	5810	5795(5)	5805.7(8.1)	$5755(^{18}_{23})$	5792.9(3.0)*
Ξ_b'	$\frac{1}{2}(\frac{1}{2}^+)$	5937	5950	5930(5)	5950.9(8.5)	$5885(^{15}_{18})$	
Ξ_b^*	$\frac{1}{2}(\frac{3}{2}^+)$	5963	5980	5959(4)	5966.1(8.3)	$5897(^{40}_{25})$	
Ω_b	$0(\frac{1}{2}^+)$	6065	6060	6052(6)	6068.7(11.1)	$6001(^{12}_{19})$	{ 6054.4(6.8)** 6165(23)*
Ω_b^*	$0(\frac{3}{2}^+)$	6088	6090	6083(6)	6083.2(11.0)	$6013(^{18}_{23})$	

† BaBar 2006; ‡ CDF 2006 (Σ_b^+); * CDF 2007; * D0 2008; ** CDF 2009

Tetraquarks in diquark-antidiquark picture

Recently charmonium-like states with exotic properies have been observed experimentally. One of posible theoretical interpretation of these states could be provided by tetraquark model

Diquark and antidiquark in colour $\bar{3}$ and 3 configurations bound by colour forces

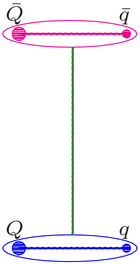
⋆ typical hadronic size

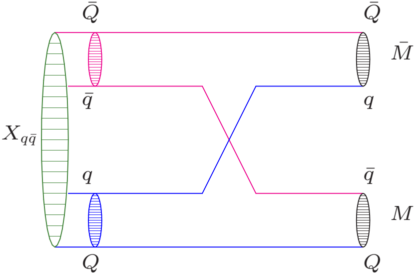
⋆ X should be split into two states $[cu][\bar{c}\bar{u}]$ and $[cd][\bar{c}\bar{d}]$ with ΔM ~few MeV

⋆ existence of charged partners $X^+ = [cu][\bar{c}\bar{d}]$, $X^- = [cd][\bar{c}\bar{u}]$

⋆ existence of tetraquarks with open $X_{s\bar{q}} = [cs][\bar{c}\bar{q}]$ and hidden $X_{s\bar{s}} = [cs][\bar{c}\bar{s}]$ strangeness

⋆ rich spectroscopy — radial and orbital excitations between diquarks





$(Q = c, b \qquad q = u, d$ or $s)$

Table: Masses of charm and bottom tetraquarks (in MeV) and possible experimental candidates.

State	Diquark	Theory		Experiment		Theory
J^{PC}	content	$cq\bar{c}\bar{q}$	$cs\bar{c}\bar{s}$	state	mass	$bq\bar{b}\bar{q}$
$1S$						
1^{++}	$(S\bar{A}+\bar{S}A)/\sqrt{2}$	3871		$X(3872)$ $X(3876)$	3871.4 ± 0.6 Belle $3875.2 \pm 0.7^{+0.9}_{-1.8}$ Belle	10492
1^{++}	$(S\bar{A}+\bar{S}A)/\sqrt{2}$		4113	$Y(4140)$	$4143.0 \pm 2.9 \pm 1.2$ CDF	
0^{++}	$A\bar{A}$		4110			
2^{++}	$A\bar{A}$	3968		$Y(3940)$	$3943 \pm 11 \pm 13$ Belle $3914.3^{+4.1}_{-3.8}$ BaBar	10534
$1P$						
1^{--}	$S\bar{S}$	4244		$Y(4260)$	$4259 \pm 8^{+2}_{-6}$ BaBar $4247 \pm 12^{+17}_{-32}$ Belle	10807
1^{-}	$S\bar{S}$	4244		$Z_2(4250)$	$4248^{+44+180}_{-29-35}$ Belle	10807
0^{-}	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	4267				10820
1^{--}	$(S\bar{A}-\bar{S}A)/\sqrt{2}$	4284		$Y(4260)$	$4284^{+17}_{-16} \pm 4$ CLEO	10824
1^{--}	$A\bar{A}$	4277				10827
1^{--}	$A\bar{A}$	4350		$Y(4360)$	$4361 \pm 9 \pm 9$ Belle 4324 ± 24 BaBar	10850
$2S$						
1^{+}	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	4431		$Z(4430)$	$4433 \pm 4 \pm 2$ Belle	10939
0^{+}	$A\bar{A}$	4434				10942
1^{+}	$A\bar{A}$	4461				10951
$2P$						
1^{--}	$S\bar{S}$	4666		$Y(4660)$ $X(4630)$	$4664 \pm 11 \pm 5$ Belle 4634^{+8+5}_{-7-8} Belle	11122

Table: Masses of light unflavoured tetraquark ground state ($\langle \mathbf{L}^2 \rangle$=0) (in MeV) and possible experimental candidates. S and A denote scalar and axial vector diquarks.

State J^{PC}	Diquark content	Theory mass	Experiment $I=0$	mass	$I=1$	mass
$(qq)(\bar{q}\bar{q})$						
0^{++}	$S\bar{S}$	596	$f_0(600)$ [σ]	400-1200		-
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	672				
0^{++}	$A\bar{A}$	1179	$f_0(1370)$	1200-1500		
1^{+-}	$A\bar{A}$	1773				
2^{++}	$A\bar{A}$	1915	$f_2(1910)$	1903(9)		
			$f_2(1950)$	1944(12)		
$(qs)(\bar{q}\bar{s})$						
0^{++}	$S\bar{S}$	992	$f_0(980)$	980(10)	$a_0(980)$	984.7(12)
1^{++}	$(S\bar{A} + \bar{S}A)/\sqrt{2}$	1201	$f_1(1285)$	1281.8(6)	$a_1(1260)$	1230(40)
1^{+-}	$(S\bar{A} - \bar{S}A)/\sqrt{2}$	1201	$h_1(1170)$	1170(20)	$b_1(1235)$	1229.5(32)
0^{++}	$A\bar{A}$	1480	$f_0(1500)$	1505(6)	$a_0(1450)$	1474(19)
1^{+-}	$A\bar{A}$	1942	$h_1(1965)$	1965(45)	$b_1(1960)$	1960(35)
2^{++}	$A\bar{A}$	2097	$f_2(2010)$	2011(70)	$a_2(1990)$	2050(45)
			$f_2(2140)$	2141(12)	$a_2(2080)$	2100(20)
$(ss)(\bar{s}\bar{s})$						
0^{++}	$A\bar{A}$	2203	$f_0(2200)$	2189(13)		-
1^{+-}	$A\bar{A}$	2267	$h_1(2215)$	2215(40)		-
2^{++}	$A\bar{A}$	2357	$f_2(2340)$	2339(60)		-

Electroweak properties of hadrons

• wave functions obatained in calculating mass spectra are used

• systematic account of relativistic effects, especially:
⋆ contributions of intermediate states with negative energy
⋆ transformation of wave function from rest to moving frame

• general method for calculation of matrix elements of local current operators was developed:
⋆ for heavy hadrons $1/m_Q$ expansion is applied
⋆ for decays of heavy hadrons with large energy transfer to light final hadrons expansion in inverse powers of recoil energy is used

• fulfillment of model independet symmetry relations following from HQET was tested and leading and subleading Isgur-Wise functions were determined

• several new symmetry relations for decays of heavy mesons to excited heavy and light mesons were derived

• decays of heavy baryons were considered in quark-diquark approximation

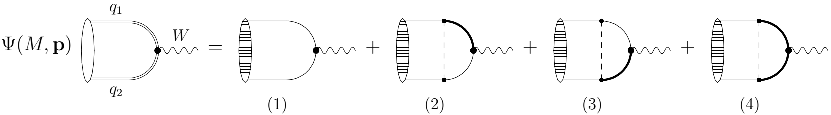
Meson decay constants

$$\begin{aligned}\langle 0|\bar{q}_1\gamma^\mu\gamma_5 q_2|P(\mathbf{K})\rangle &= i f_P K^\mu,\\ \langle 0|\bar{q}_1\gamma^\mu q_2|V(\mathbf{K},\varepsilon)\rangle &= f_V M_V \varepsilon^\mu,\end{aligned}$$

K – meson momentum, ε^μ – polarisation vector, M_V – vector meson mass.

$$\left\langle 0|J^W_\mu|M(\mathbf{K})\right\rangle = \int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left\{\gamma_\mu(1-\gamma_5)\Psi(M,p)\right\},$$

$\Psi(M,p)$ – two-particle Bethe-Salpeter wave function



- completely relativistic treatment of light quarks
- contributions of negative-energy states are taken into account (bold lines)

$$f_{P,V} = f^{(1)}_{P,V} + f^{(2+3)}_{P,V} + f^{(4)}_{P,V},$$

contributions $f^{(2+3)}_{P,V}$ and $f^{(4)}_{P,V}$ are new

$$\Psi(M,\mathbf{p}) = \int \frac{dp^0}{2\pi}\Psi(M,p)$$

Table: Different contributions to the pseudoscalar and vector decay constants of light and heavy-light mesons (in MeV).

Constant	f_M^{NR}	$f_M^{(1)}$	$f_M^{(2+3)} + f_M^{(4)}$	$(f_M^{(2+3)} + f_M^{(4)})/f_M^{(1)}$	f_M	Exp.
f_π	1290	515	-391	-76%	124	130.4(2)
f_ρ	490	402	-183	-46%	219	
f_K	783	353	-198	-56%	155	155.5(9)
f_{K^*}	508	410	-174	-42%	236	
f_ϕ	511	415	-170	-41%	245	
f_D	376	275	-41	-15%	234	208(9)
f_{D^*}	391	334	-24	-7%	310	
f_{D_s}	436	306	-38	-12%	268	269(9)
$f_{D_s^*}$	447	367	-52	-14%	315	
f_B	259	210	-21	-10%	189	227(52)
f_{B^*}	280	235	-16	-7%	219	
f_{B_s}	300	238	-20	-8%	218	
$f_{B_s^*}$	316	264	-13	-5%	251	
f_{B_c}	538	433	-8	-1.8%	425	
$f_{B_c^*}$	545	503	-4	-0.8%	499	

$$f_{P,V}^{\mathrm{NR}} = \sqrt{\frac{12}{M_{P,V}}}\,|\Psi_{P,V}(0)|$$

Table: Pseudoscalar and vector decay constants of light mesons (in MeV).

Constant	RQM	Godfrey	Maris	Koll	He	Ali Khan	Milc	Experiment
f_π	124	180	131	219	138	126.6(6.4)	129.5(3.6)	130.70(10)(36)
f_K	155	232	155	238	160	152.0(6.1)	156.6(3.7)	155.5(1.0)(2)
f_ρ	219	220	207		238	239.4(7.3)		$\begin{cases} 220(2)^{*} \\ 209(4)^{**} \end{cases}$
f_{K^*}	236	267	241		241	255.5(6.5)		$217(5)^{\dagger}$
f_ϕ	245	336	259			270.8(6.5)		$229(3)^{\ddagger}$

* obtained using experimental value for $\Gamma_{\rho^0\to e^+e^-}$.
** obtained using experimental value for $\Gamma_{\tau\to\rho\nu_\tau}$.
† obtained using experimental value for $\Gamma_{\tau\to K^*\nu_\tau}$.
‡ obtained using experimental value for $\Gamma_{\phi\to e^+e^-}$.

Table: Pseudoscalar decay constants of heavy-light mesons (in MeV).

Constant	Quark Model		Lattice		QCD sum rules			Experiment
	RQM	Cvetic	ETMC	Milc	Narison	Penin	Jamin	PDG
f_D	234	230(25)	197(9)	207(11)	203(20)	195(20)		207.6(9.6)
f_{D_s}	268	248(27)	244(8)	249(11)	235(24)			269.6(8.3)
f_{D_s}/f_D	1.15	1.08(1)	1.24(3)	1.20(3)	1.15(4)			
f_B	189	196(29)		195(11)	203(23)	206(20)	210(19)	227(52)
f_{B_s}	218	216(32)		243(11)	236(30)		244(21)	
f_{B_s}/f_B	1.15	1.10(1)		1.24(4)	1.16(4)		1.16	

Electromagnetic form factors of light mesons

$$\langle M(P_F)|J_\mu|M(P_I)\rangle = F_P(Q^2)(P_I+P_F)_\mu, \qquad Q^2=-(P_F-P_I)^2$$

Conservation of electric charge $\longrightarrow$ normalisation condition: $F_P(0)=1$

- completely relativistic calculation in the wide range of space-like momenta $Q^2 \geq 0$
- contributions of negative-energy states are taken into account
- calculated pion form factor for large Q^2 has the asymptotic behaviour

$F_\pi(Q^2) \sim \alpha_s(Q^2)/Q^2$ predicted by quark counting rules and perturbative QCD

Mean charge radius squared of pseudoscalar meson ($P=\pi, K$):

$$\langle r^2\rangle_P = -6\left[\frac{\mathrm{d}F_P(Q^2)}{\mathrm{d}Q^2}\right]_{Q^2=0}.$$

Table: Charge radii of pseudoscalar mesons.

Charge radius	our	Godfrey	Maris	He	Lattice	Experiment
$\sqrt{\langle r^2\rangle_\pi}$ (fm)	0.66	0.66	0.67	0.63	0.63 ± 0.1	0.672 ± 0.08
$\sqrt{\langle r^2\rangle_{K^\pm}}$ (fm)	0.57	0.59	0.62	0.60		0.560 ± 0.031
$\langle r^2\rangle_{K^0}$ (fm^2)	-0.072	-0.09	-0.086	-0.062		-0.076 ± 0.018

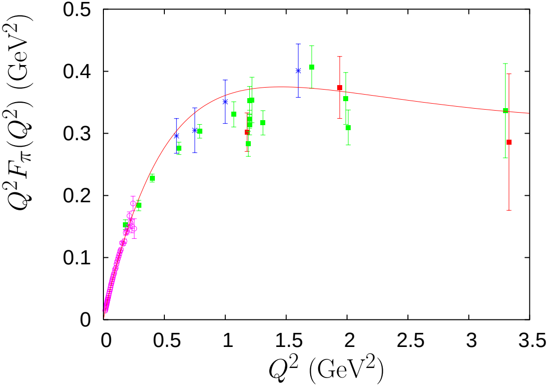


Figure: The product of Q^2 and form factor of charged pion in comparison with experimental data.

Two-photon decays of heavy quarkonia

Table: Two-photon decay widths of pseudoscalar ($^1S_0$), scalar ($^3P_0$) and tensor ($^3P_2$) states of heavy quarkonia (in keV).

Meson	Theory							Experiment
	RQM	Munz	Gupta	Schuler	Huang	Ackleh	Crater	PDG
$\eta_c(1^1S_0)$	5.5	3.5	10.94	7.8	5.5	4.8	6.18	7.2(2.1)
$\eta_c'(2^1S_0)$	1.8	1.38		3.5	2.1	3.7	1.95	1.3(6)
$\chi_{c0}(1^3P_0)$	2.9	1.39	6.38	2.5	5.32		3.34	2.4(4)
$\chi_{c0}'(2^3P_0)$	1.9	1.11						
$\chi_{c2}(1^3P_2)$	0.50	0.44	0.57	0.28	0.44		0.436	0.52(5)
$\chi_{c2}'(2^3P_2)$	0.52	0.48						
$\eta_b(1^1S_0)$	0.35	0.22	0.46	0.46	0.45	0.17		
$\eta_b'(2^1S_0)$	0.15	0.11		0.20	0.21	0.13		
$\eta_b''(3^1S_0)$	0.10	0.084						
$\chi_{b0}(1^3P_0)$	0.038	0.024	0.080	0.043				
$\chi_{b0}'(2^3P_0)$	0.029	0.026						
$\chi_{b2}(1^3P_2)$	0.008	0.0056	0.008	0.0074				
$\chi_{b2}'(2^3P_2)$	0.006	0.0068						

Radiative decays

(a) Magnetic dipole ($M1$) transitions ($\Delta S = 1$, $\Delta L = 0$).

Table: $M1$ radiative decay widths in charmonium.

Decay	ω MeV	$\Gamma^{\rm NR}$ keV	Γ^V keV	Γ^S keV	Γ keV	$\Gamma^{\rm exp}$ keV
$J/\Psi \to \eta_c\gamma$	115	2.73	1.95	3.13	1.05	1.58 ± 0.41
$\Psi' \to \eta_c'\gamma$	49	0.20	0.13	0.11	0.15	< 0.62
$\Psi' \to \eta_c\gamma$	639	0.23	0.61	0.35	0.95	1.05 ± 0.19
$\eta_c' \to J/\Psi\gamma$	500	0.33	0.88	0.47	1.41	

Table: $M1$ radiative decay widths in bottomonium.

Decay	ω MeV	$\Gamma^{\rm NR}$ eV	Γ^V eV	Γ^S eV	Γ eV	BR (10^{-4})	$BR^{\rm exp}$ (10^{-4})
$\Upsilon \to \eta_b\gamma$	60	9.7	8.7	12.2	5.8	1.1	
$\Upsilon' \to \eta_b'\gamma$	33	1.6	1.45	1.50	1.40	0.43	
$\Upsilon'' \to \eta_b''\gamma$	27	0.9	0.8	0.8	0.8	0.39	
$\Upsilon' \to \eta_b\gamma$	604	1.3	3.4	1.3	6.4	2.1	$4.2^{+1.1}_{-1.0} \pm 0.9$
$\eta_b' \to \Upsilon\gamma$	516	2.4	6.3	2.5	11.8		
$\Upsilon'' \to \eta_b\gamma$	911	2.5	6.2	3.1	10.5	5.1	$4.8 \pm 0.5 \pm 1.2$
$\eta_b'' \to \Upsilon\gamma$	831	5.8	14.3	7.1	24.0		
$\Upsilon'' \to \eta_b'\gamma$	359	0.2	0.6	0.1	1.5	0.74	< 6.2
$\eta_b'' \to \Upsilon'\gamma$	301	0.4	1.1	0.2	2.8		

(b) Electric dipole ($E1$) transitions ($\Delta L = 1$, $\Delta S = 0$).

Table: $E1$ radiative decay widths in charmonium.

Decay	ω MeV	$\Gamma^{\rm NR}$ keV	Γ^{V} keV	Γ^{S} keV	Γ keV	$\Gamma^{\rm exp}$ (PDG) keV
$2\,^3S_1 \to 1\,^3P_0\gamma$	259	51.7	34.6	44.0	26.3	29.1 ± 1.8
$2\,^3S_1 \to 1\,^3P_1\gamma$	171	44.9	30.1	38.3	22.9	28.4 ± 2.1
$2\,^3S_1 \to 1\,^3P_2\gamma$	128	30.9	22.9	28.1	18.2	26.8 ± 1.9
$2\,^1S_0 \to 1\,^1P_1\gamma$	120	54	39	39	39	
$1\,^3P_0 \to 1\,^3S_1\gamma$	305	161	151	184	121	119 ± 16
$1\,^3P_1 \to 1\,^3S_1\gamma$	389	333	285	305	265	293 ± 30
$1\,^3P_2 \to 1\,^3S_1\gamma$	430	448	309	292	327	384 ± 37
$1\,^1P_1 \to 1\,^1S_0\gamma$	504	723	560	560	560	
$1\,^3D_1 \to 1\,^3P_0\gamma$	361	423	344	334	355	199 ± 32
$1\,^3D_1 \to 1\,^3P_1\gamma$	277	142	127	120	135	79 ± 20
$1\,^3D_1 \to 1\,^3P_2\gamma$	234	5.8	6.2	5.6	6.9	<24
$1\,^3D_2 \to 1\,^3P_1\gamma$	291	297	215	215	215	
$1\,^3D_2 \to 1\,^3P_2\gamma$	248	62	55	51	59	
$1\,^3D_3 \to 1\,^3P_2\gamma$	250	252	163	170	156	
$1\,^1D_2 \to 1\,^1P_1\gamma$	275	335	245	245	245	

Table: $E1$ radiative decay widths in bottomonium.

Decay	ω MeV	$\Gamma^{\rm NR}$ keV	Γ^{V} keV	Γ^{S} keV	Γ keV	$\Gamma^{\rm exp}$ keV
$2\,^3S_1 \to 1\,^3P_0\gamma$	162	1.65	1.64	1.66	1.62	1.22 ± 0.23
$2\,^3S_1 \to 1\,^3P_1\gamma$	130	2.57	2.48	2.51	2.45	2.2 ± 0.3
$2\,^3S_1 \to 1\,^3P_2\gamma$	109	2.53	2.49	2.52	2.46	2.3 ± 0.3
$2\,^1S_0 \to 1\,^1P_1\gamma$	98	3.25	3.09	3.09	3.09	
$1\,^3P_0 \to 1\,^3S_1\gamma$	391	29.5	30.6	31.3	29.9	
$1\,^3P_1 \to 1\,^3S_1\gamma$	422	37.1	37.0	37.4	36.6	
$1\,^3P_2 \to 1\,^3S_1\gamma$	442	42.7	39.8	39.3	40.2	
$1\,^1P_1 \to 1\,^1S_0\gamma$	480	54.4	52.6	52.6	52.6	
$3\,^3S_1 \to 2\,^3P_0\gamma$	123	1.65	1.51	1.52	1.49	1.20 ± 0.25
$3\,^3S_1 \to 2\,^3P_1\gamma$	100	2.65	2.43	2.45	2.41	2.56 ± 0.47
$3\,^3S_1 \to 2\,^3P_2\gamma$	86	2.89	2.69	2.71	2.67	2.7 ± 0.52
$3\,^1S_0 \to 2\,^1P_1\gamma$	73	3.07	2.78	2.78	2.78	
$3\,^3S_1 \to 1\,^3P_0\gamma$	484	0.124	0.040	0.054	0.027	0.061 ± 0.028
$3\,^3S_1 \to 1\,^3P_1\gamma$	453	0.307	0.097	0.134	0.067	< 0.035
$3\,^3S_1 \to 1\,^3P_2\gamma$	433	0.445	0.141	0.195	0.097	< 0.40
$3\,^1S_0 \to 1\,^1P_1\gamma$	427	0.770	0.348	0.348	0.348	

Semileptonic heavy-to-heavy meson decays

Semileptonic B meson decays:
$B \to D^{(*)} e\nu$ – to ground state D mesons
$B \to D^{(**)} e\nu$ – to orbitally excited $D(1P_J)$ mesons
$B \to D'^{(*)} e\nu$ – to radially excited $D(2S)$ mesons

$$\begin{aligned}
\frac{\langle D(v')|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{M_D M_B}} &= h_+(v+v')^\mu + h_-(v-v')^\mu, \\
\frac{\langle D^*(v',\epsilon)|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{M_{D^*} M_B}} &= i h_V \varepsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha v'_\beta v_\gamma, \\
\frac{\langle D^*(v',\epsilon)|\bar{c}\gamma^\mu\gamma_5 b|B(v)\rangle}{\sqrt{M_{D^*} M_B}} &= h_{A_1}(w+1)\epsilon^{*\mu} - (h_{A_2} v^\mu + h_{A_3} v'^\mu)(\epsilon^* \cdot v),
\end{aligned}$$

$w = v \cdot v'$ is scalar product of meson four-velocities

- $1/m_Q$ expansion. Leading Isgur-Wise function ($m_Q \to \infty$) for decay $B \to D^{(*)} e\nu$

$$\xi(w) = \sqrt{\frac{2}{w+1}} \lim_{m_Q\to\infty} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_D\left(\mathbf{p} + 2\epsilon_q(p)\sqrt{\frac{w-1}{w+1}}\mathbf{e_\Delta}\right)\Psi_B(\mathbf{p})$$

$$h_+(w) = h_{A_1}(w) = h_{A_3}(w) = h_V(w) = \xi(w); \quad h_-(w) = h_{A_2}(w) = 0.$$

Obtained mean value of Cabibbo-Kobayashi-Maskava matrix element $|V_{cb}|$:

$$|V_{cb}| = 0.0385 \pm 0.0015.$$

PDG value (exclusive)

$$|V_{cb}| = 0.0386 \pm 0.0013$$

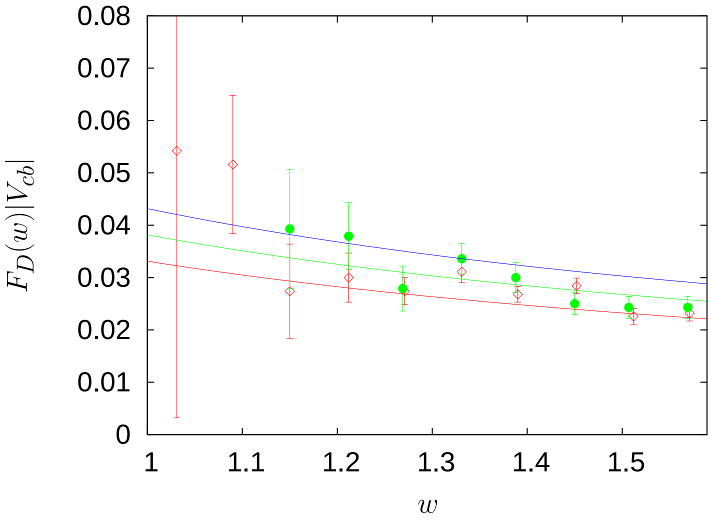


Figure: Comparison of experimental data and predictions of our model for product $F_D(w)|V_{cb}|$. Green dots represent CLEO data and red diamonds show Belle data. Solid lines are predictions of our model for $|V_{cb}| = 0.044,\ 0.039,\ 0.034$ (blue, green, red).

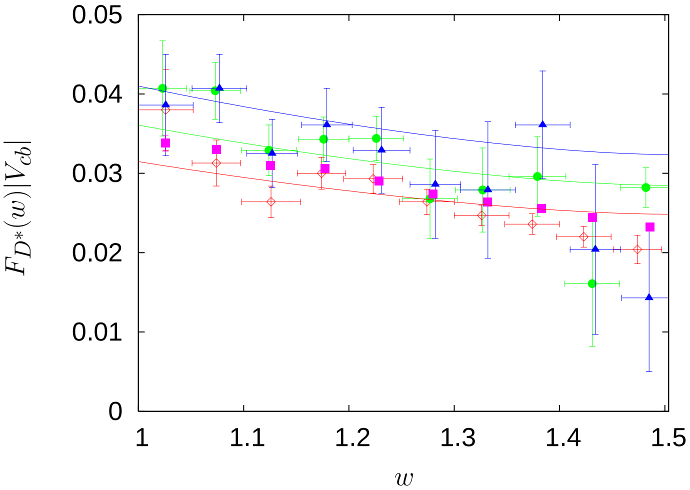


Figure: Comparison of experimental data and predictions of our model for product $F_{D^*}(w)|V_{cb}|$. Dots show CLEO data for $B^+ \to D^{*0} l^- \nu$, triangles – CLEO data for $B^0 \to D^{*+} l^- \nu$, diamonds – Belle data, squares – BaBar data. Solid lines show predictions of our model for $|V_{cb}| = 0.044,\ 0.039,\ 0.034$ (blue, green, red).

Semileptonic heavy-to-light meson decays

$B \to \pi(\rho) e\nu$

Determination of Cabibbo-Kobayashi-Maskawa matrix element $|V_{ub}|$:

- from partial and total semileptonic decay widths

$$\begin{array}{ll} B \to \pi e\nu & |V_{ub}| = (4.02 \pm 0.10) \times 10^{-3}, \\ B \to \rho e\nu & |V_{ub}| = (3.33 \pm 0.20) \times 10^{-3}, \end{array}$$

- from comparison of branching ratios

$$\begin{array}{ll} B \to \pi e\nu & |V_{ub}| = (4.05 \pm 0.20) \times 10^{-3}, \\ B \to \rho e\nu & |V_{ub}| = (3.38 \pm 0.30) \times 10^{-3}, \end{array}$$

- $|V_{ub}|$ from $B \to \rho e\nu$ is $\sim 16\%$ smaller than $|V_{ub}|$ from $B \to \pi e\nu$
- from recent CLEO data for branching ratio

$$BR(B^0 \to \rho^+ e^- \nu)^{\rm exp} = (2.91 \pm 0.54) \times 10^{-4} \qquad |V_{ub}| = (3.81 \pm 0.35) \times 10^{-3}$$

close to value of $|V_{ub}|$ from $B \to \pi e\nu$

- obtained mean value

$$|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}.$$

PDG value (exclusive)

$$|V_{ub}| = (3.5^{+0.6}_{-0.5}) \times 10^{-3}$$

Rare radiative B meson decays

$B \to K^{(*)}_{(J)}\gamma$

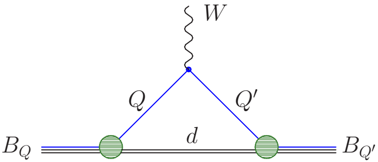
- induced by flavour changing neutral currents
- described by one-loop diagrams (penguins)

Table: Theoretical predictions and experimental data for branching ratios of rare radiative decays of B mesons ($\times 10^{-5}$)

	RQM	Altomari	Veseli	Safir	Lee	Cheng	Exp.
$BR(B \to K^*(892)\gamma)$	4.4(5)	1.35	4.71(1.79)	3.4(1.4)		3.27(74)	$4.01(20)^a$ $4.03(26)^b$ $4.2(6)^c$
$B \to K_0^*(1430)\gamma$	forbidden						
$BR(B \to K_1(1270)\gamma)$	0.59(11)	1.1	1.20(44)	0.69(28)	0.828	0.77(11)	$< 5.8^a$ $4.3(1.3)^b$
$BR(B \to K_1(1400)\gamma)$	0.86(15)	0.7	0.58(26)	0.31(14)	0.393	0.08(4)	$< 1.5^{a,b}$
$BR(B \to K_2^*(1430)\gamma)$	1.7(4)	1.8	1.73(80)	1.7(7)		1.48(30)	$1.24(24)^a$ $1.4(4)^b$ $1.7(6)^c$

Experimental data for decays of: ^{a} B^0; ^{b} B^+; ^{c} mixture of $B^\pm/B^0$ mesons.

Semileptonic decays of heavy baryons



Form factors of heavy baryons with scalar diquark

Hadronic matrix elements $\Lambda_Q \to \Lambda_{Q'} e\nu$ $(w = v \cdot v')$:

$$\langle \Lambda_{Q'}(v',s')|V^\mu|\Lambda_Q(v,s)\rangle = \bar{u}_{\Lambda_{Q'}}(v',s')\Big[F_1(w)\gamma^\mu + F_2(w)v^\mu + F_3(w)v'^\mu\Big]u_{\Lambda_Q}(v,s),$$
$$\langle \Lambda_{Q'}(v',s')|A^\mu|\Lambda_Q(v,s)\rangle = \bar{u}_{\Lambda_{Q'}}(v',s')\Big[G_1(w)\gamma^\mu + G_2(w)v^\mu + G_3(w)v'^\mu\Big]\gamma_5 u_{\Lambda_Q}(v,s)$$

- In the limit $m_Q \to \infty$ (leading order) according to HQET

$$F_1(w) = G_1(w) = \zeta(w), \qquad F_2(w) = F_3(w) = G_2(w) = G_3(w) = 0.$$

- In the first (subleading) order of $1/m_Q$ expansion in HQET

$$
\begin{aligned}
F_1(w) &= \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)[2\chi(w)+\zeta(w)], \\
G_1(w) &= \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)\left[2\chi(w)+\frac{w-1}{w+1}\zeta(w)\right], \\
F_2(w) = G_2(w) &= -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w), \qquad F_3(w) = -G_3(w) = -\frac{\bar\Lambda}{2m_Q}\frac{2}{w+1}\zeta(w).
\end{aligned}
$$

In the first order of $1/m_Q$ expansion in our model

$$
\begin{aligned}
F_1(w) &= \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)[2\chi(w)+\zeta(w)] \\
&\quad +4(1-\varepsilon)(1+\kappa)\left[\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1} - \frac{\bar\Lambda}{2m_Q}(w+1)\right]\chi(w), \\
G_1(w) &= \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)\left[2\chi(w)+\frac{w-1}{w+1}\zeta(w)\right] - 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_Q}w\chi(w), \\
F_2(w) &= -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w) - 4(1-\varepsilon)(1+\kappa)\left[\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1} + \frac{\bar\Lambda}{2m_Q}w\right]\chi(w), \\
G_2(w) &= -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w) - 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1}\chi(w), \\
F_3(w) &= -G_3(w) = -\frac{\bar\Lambda}{2m_Q}\frac{2}{w+1}\zeta(w) + 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_Q}\chi(w).
\end{aligned}
$$

$\Longrightarrow (1-\varepsilon)(1+\kappa) = 0$ ($\kappa = -1$ in our model)!

Prediction for branching ratio ($|V_{cb}| = 0.041$, $\tau_{\Lambda_b} = 1.23 \times 10^{-12}$s)

$$Br^{\rm theor}(\Lambda_b \to \Lambda_c e\nu) = 6.9\%$$

Experiment

$$Br^{\rm exp}(\Lambda_b \to \Lambda_c e\nu) = \begin{cases} \left(5.0^{+1.1\,+1.6}_{-0.8\,-1.2}\right)\% & \text{DELPHI} \\ \left(8.1 \pm 1.2^{+1.1}_{-1.6} \pm 4.3\right)\% & \text{CDF} \end{cases}$$

$$Br^{\rm exp}(\Lambda_b \to \Lambda_c e\nu + \text{anything}) = (9.9 \pm 2.6)\%. \qquad \text{PDG}$$

Table: Comparison of theoretical predictions for semileptonic decay widths Γ (in $10^{10}\mathrm{s}^{-1}$) of bottom baryons.

Decay	RQM	Singleton NRQM	Cheng NRQM	Körner NRQM	Ivanov RTQM	Ivanov BS	Cardarelli LF	Albertus NRQM	Huang sum rule
$\Lambda_b \to \Lambda_c e\nu$	5.64	5.9	5.1	5.14	5.39	6.09	5.08 ± 1.3	5.82	5.4 ± 0.4
$\Xi_b \to \Xi_c e\nu$	5.29	7.2	5.3	5.21	5.27	6.42	5.68 ± 1.5	4.98	
$\Sigma_b \to \Sigma_c e\nu$	1.44	4.3			2.23	1.65			
$\Xi'_b \to \Xi'_c e\nu$	1.34								
$\Omega_b \to \Omega_c e\nu$	1.29	5.4	2.3	1.52	1.87	1.81			
$\Sigma_b \to \Sigma_c^* e\nu$	3.23				4.56	3.75			
$\Xi'_b \to \Xi_c^* e\nu$	3.09								
$\Omega_b \to \Omega_c^* e\nu$	3.03			3.41	4.01	4.13			

Conclusions

- Constituent quark model provides reliable pattern for describing hadron properties
- Quasipotential method embodying QCD motivated quark interaction is the effective means of constructing relativistic quark model
- Long-range quark interaction in hadrons is the mixture of scalar and vector confining potentials
- Chromomagnetic quark interaction vanishes at large distances in accord with flux tube model
- One must consistently take into account relativistic effects including contributions of intermediate negative-energy states and relativistic transformations of hadron wave functions
- For describing heavy hadrons it is permissible to apply the expansion in relative velocities (v/c) and in inverse masses ($1/m_Q$) of heavy quarks
- Heavy quark symmetry imposes strong constraints on the model parameters and essentially simplifies calculations
- Light quarks should be treated completely relativistically
- Diquark concept affords good approximation for describing properties of heavy baryons and tetraquarks

"My conclusion is that if you want to know the mass of a particle and if you have little time (in years!) and little money you should forget all your prejudices and use potential models. This is, in fact, even true to a large extent for systems containing light quarks, which is still more mysterious."

André Martin (CERN)

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