Do 15^{00} – 16^{30} VL

Mitschrift: Laura Orphal

40498 Applied Photonics [P24.4.a,P35.2]]

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Topics:

Fourier optics
propagation of light in free space, scalar wave propagation and transfer functions,
Fresnel- and Fraunhofer transformations, optical elements, optical spatial filtering, image formation

Modern Microscopy
Amplitude contrast, Zernike phase contrast, dark field, fluorescence microscopy, X-ray microscopy, super-resolution microscopy

Coupled wave theory
Volume gratings, mode coupling, examples for volume gratings

Waves + periodic Structures = photonic crystals (= semiconductors)
photonic crystals, 3D photonic crystals, Bragg reflections, optical matrix method

Fiber optics
Single mode/ multimode fibers, material- & waveguide-dispersion, propagation of short pulses

Optical communications
Switches, modulators, LCD, MEMS, AOM, Shannon theorem, multiplexing

Semiconductors
materials: IV, III-V, II-VI, Bandgap engineering, (in-)direct bandgap, effective mass, density of states + Fermi distribution, organic / molecular semiconductors, HOMO-LUMO

Semiconductor devices
detectors:
photomultiplier, photodiode, CCD, APD, dynamic range, sensitivity, noise, Poisson statistics
(Laser-) Diodes:
emission condition, quantum-heterostructures, DFB laser, OLED
solar cells:
Si-photovoltaics, organic & perovskite solar cells, chemical tuning of optical properties

Sensors
Raman / IR / fluorescence-label / bio-molecular recognition, fiber sensors, plasmonics, surface enhancement (SERS)

[redacted] Laura Orphal

20. 10. 2016

Applied Photonics

Q1 C

Q2 B Nöther Theorem

Q3 A $E = \frac{hc}{\lambda}$ $\lambda = \frac{hc}{E}$

Q4 D

Q5 C

$\lambda\,[nm] = \frac{1240}{E\,[eV]}$

key numbers

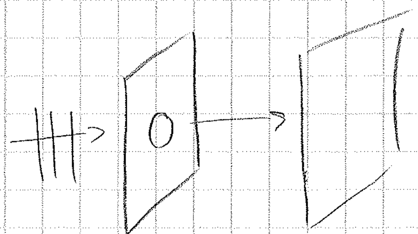
$hc = 1240$

room temperature $= 26\,meV$

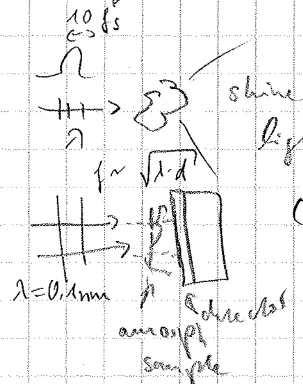
24. 10. 2016
Gerd Schneider (HZB)

X-ray spectroscopy : short wavelength

synchrotron: small source, low convergence



Fourier Optics
→ Fraunhofer, Fresnel



shine monochromatic light on sample

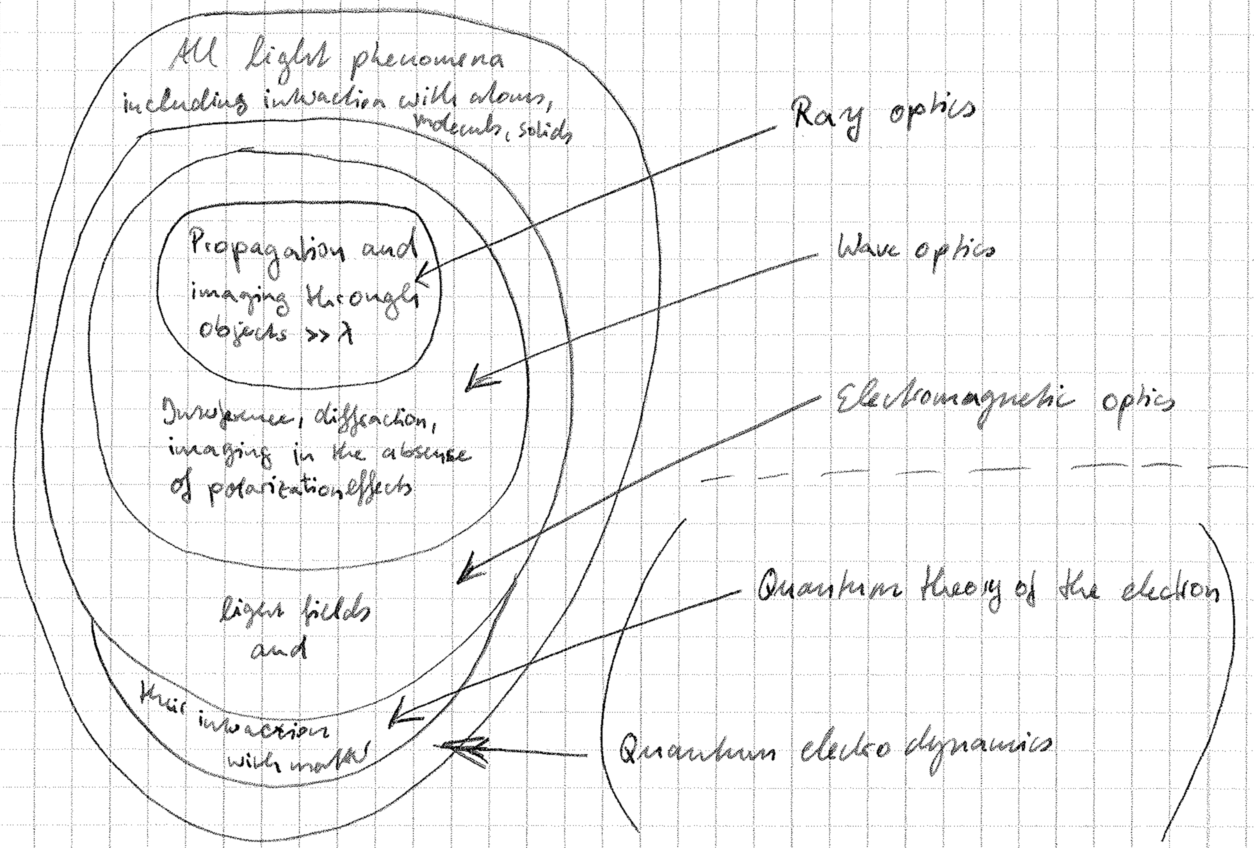
crystal → spots
amorph →

Contact microscopy (atomic resolution)
grating → diffraction

X-ray : materials have all the same refractive index (no scattering)

27.10.2016

Modelling light and its interaction with matter



(I Ray optics: propagation of light rays through simple optical components and systems (geometrical optics))

II Wave optics: propagation of light waves through optical components and systems

⇒ Scalar wave equation; Applications: Fourier optics: imaging and XCT, optical beam: propagation, filtering, focusing

III Electromagnetic theory of light: description of light waves in terms of electric and magnetic fields

Postulates: Maxwell's equations, electromagnetic power flow: poynting vector. Derivation of wave fields from electromagnetic theory. Approximations: paraxial electromagnetic waves, linear and nonlinear polarizability of matter

IV Semiclassical theory of light-matter interactions: EM theory of light and quantum theory of the electron. Phenomena / Applications: polarizability of matter, light induced

atomic transitions, absorption and stimulated emission of light
principles of lasers

V Quantum-Optics: description of light in terms of photons

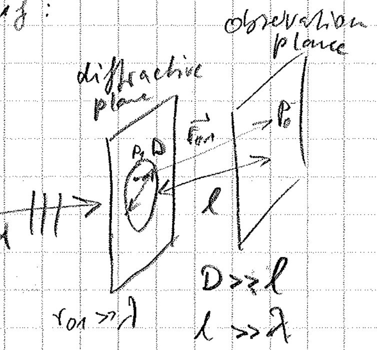
lifetime of excited atomic states, spontaneous emission of light, photon detectors, quantum noise

Literature: E. A. Saleh, M. C. Teich - Fundamentals of Photonics, John Wiley and S.

Basics of image formation by diffraction theory (Wave optics)

In the following we consider scalar electrical waves. The function $U(\vec{r},t)$ describes the scalar amplitude of the transversal wave of the electrical field. The coupling given by the Maxwell equations of the electrical and magnetic field vectors is neglected. This approach is valid, if:

1) the diffractive aperture itself and
2) the distance between the observation plane and the diffractive plane is large compared to the wavelength of the used light



Under these conditions, all amplitude distributions have to be solutions of the scalar wave equation (in free space propagation):

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)U(\vec{r},t)=0 \qquad (1)$$

For monochromatic light we can separate the amplitude function $U(\vec{r},t)$ into spatial and time-dependent factors:

$$U(\vec{r},t)=U(\vec{r})\cdot e^{-2\pi i\nu t} \qquad (2)$$

Introducing (2) into (1), we get the time independent wave equation. (→ Helmholtz equation):

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k^2\right)U(\vec{r},t)=0 \qquad (3)$$

with $\vec{k}=\frac{2\pi\nu}{c}=\frac{2\pi}{\lambda}$

diffr. pattern
→ stationary solution

$U(\vec{r})$ is in general a complex function in space (x, y, z) and describes stationary solutions (time-independent). Because the wave equation is linear, all solutions have to fullfill the superposition principle.

If $U_1(\vec{r}), U_2(\vec{r}) \dots U_N(\vec{r})$ are all different solutions of the ~~Helmholtz~~ wave equation, any arbitrary linear combination of $U_1(\vec{r}) \dots U_N(\vec{r})$ is also solution of the Helmholtz equation.

What could be a solution for $U(\vec{r})$?

In diffraction theory, we consider the amplitude distribution in a plane behind an aperture Σ, which is illuminated by an incident wave field. This is a boundary condition problem.

For the special case of a planar screen, we obtain for the amplitude distribution $U(\vec{r})$ in a point P_0, as a solution, the

<u>Rayleigh-Sommerfeld equation</u> : Σ - aperture gives integral limits

wirkliche Amplitude (Blickwinkel)

$$U(\vec{r}_0) = \frac{1}{i\lambda} \iint_{\Sigma} U(\vec{r}_1) \frac{e^{ikr_{01}}}{r_{01}} \cos(\vec{n}, \vec{r}_{01}) \underbrace{ds}_{dx\,dy} \qquad \vec{r}_{01} = \vec{r}_1 - \vec{r}_0$$

The amplitude $U(\vec{r}_0)$ in the point P_0 is an area integral over the aperture Σ and the amplitude distribution $U(\vec{r}_1)$.

$\frac{e^{ikr_{01}}}{r_{01}}$ is an attenuation of spherical waves emitted from each of the point (Huygens principle)

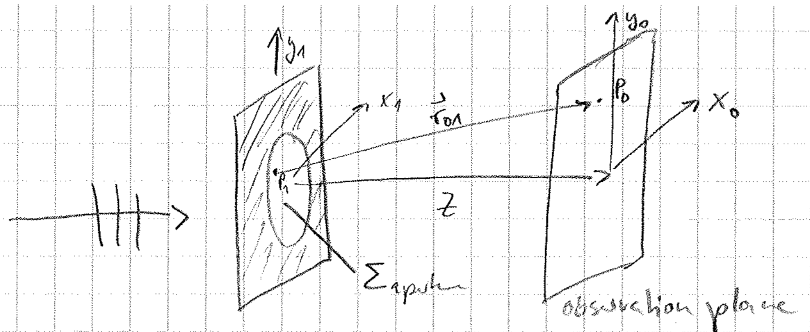
$\cos(\underbrace{\vec{n}}_{\text{normal to aperture}}, \vec{r}_{01})$ - projection term of the wave intensity

Rayl. Som. equ. is the exact formulation of Huygens principle.

The incident wave on the aperture Σ propagates as spherical waves emerging from each infinitely small area ds. In addition we have the projection factor $(\cos(\vec{n}, \vec{r}))$, which takes into account the propagation angle towards the observation point.

31.10.2016

basics of Fourier optics



The planar screen (left side) has coordinates $(x_1, y_1, z=0)$, the observation plane (right side) is parallel to the planar screen at the distance z with coordinates $(x_0, y_0, \underset{\text{const.}}{z})$.

The Rayleigh-Sommerfeld function simplifies:

$$U(x_0, y_0, z=\text{const}) = \frac{1}{i\lambda} \iint U(x_1, y_1) \frac{\exp(ikr_{01})}{r_{01}} \left(\frac{z}{r_{01}}\right) dx_1 dy_1 \qquad (1)$$

same as $\cos(\vec{n}, \vec{r}_{01}) = \cos\alpha$ ($\vec{n}$: normal to aperture) Lambert's cosine law

with $r_{01} \sqrt{(x_1-x_0)^2 + (y_1-y_0)^2 + \underset{\text{const}}{z}}$

difficult to solve integral because of the fct. r_{01}

If we treat the distance z as a parameter, we can consider the integral (1) as follows:

From the input function $U(x_1, y_1)$, we obtain by convolution with the invariant impulse function:

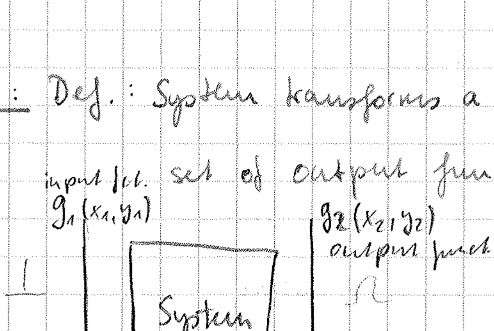
$$h(x_1-x_0, y_1-y_0, z) = \frac{1}{i\lambda} \cdot \frac{\exp(ikr_{01})}{r_{01}} \frac{z}{r_{01}}$$

translation invariant

the output function: $U(x_0, y_0, z)$

The arrangement of the screen and the parallel observation plane is an invariant linear system.

<u>linear system:</u> Def.: System transforms a set of input functions into a set of output functions.



convolution $\longrightarrow$

$\longleftarrow$ deconvolution

problem: noise

Formally, the image formation is given by an operator

$$g_2(x_2, y_2) = \overset{\text{operator}}{\hat{L}}\{g_1(x_1, y_1)\} \qquad \hat{H}\psi = E\psi$$

linear operators fulfilling Helmholtz equ. are linear: (not apply for non-linear optics, high fields)

$$L\{a g_1(x_1, y_1) + b h_1(x_1, y_1)\} = a L\{g_1(x_1, y_1)\} + b L\{h_1(x_1, y_1)\}$$

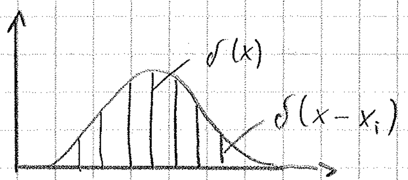
In optics the linear operators are often are integral transforms.

$$\underbrace{g_2(x_2, y_2)}_{\text{output measured}} = \iint h(x_2, y_2; x_1, y_1)\, \underbrace{g(x_1, y_1)}_{\text{input fct. (object)}}\, dx_1\, dy_1$$

Here $h(x_2, y_2, x_1, y_1)$ is only given by the optical system

$h \rightarrow$ fixed by the system, does not change by the object!

In general, objects can be described as a linear combination of δ-functions (infinitely sharp, linear weighting parameter)



which is an important example for elementary functions.

According to the translation theorem each function g_1 can be considered as weighted linear combinations of shifted δ-functions.

$$g_1(x_1, y_1) = \iint \underbrace{g_1(\xi, \eta)}_{\text{weighting function}}\, \delta(x_1 - \xi, y_1 - \eta)\, d\xi\, d\eta$$

If we consider the local δ-function at the point $x_1 = \xi$, $y_1 = \eta$, we obtain:

$$g_1(x_2, y_2) = \iint h(x_2, y_2; x_1 y_1)\, \delta(x_1 - \xi, y_1 - \eta)\, dx_1\, dy_1$$

$$= h(x_2, y_2; \xi, \eta) \qquad \text{measure broadening of original } \delta\text{-function}$$

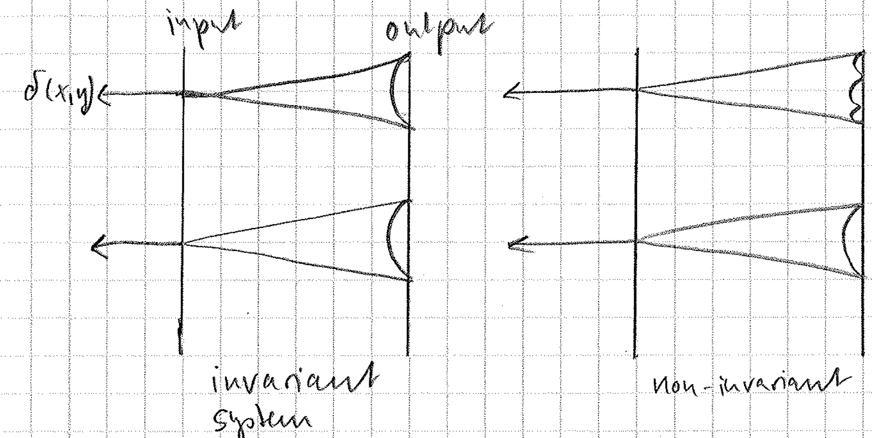
which is the integral core also named "point image".

So h is the <u>point spread function</u>

An important special case represent invariant linear systems.

For these impulse response function depends on the differences of the coordinates $x_2 - x_1$, $y_2 - y_1$.

This simplifies to $h(x_2, y_2; x_1, y_1) = h(x_2 - x_1, y_2 - y_1)$



$\delta(x,y) \longrightarrow \text{image}(x_2, y_2)$

↑ point spread function

linear invariant systems: shifted input-functions of the same shape yield also shifted output-function of the same shape (i.e. independent of the position on the screen)

For invariant linear systems we obtain with $g_2(x_2, y_2) = \iint h(x_2, y_2; x_1, y_1)\, g(x_1, y_2)\, dx_1\, dy_1$

and $h(x_2, y_2; x_1, y_1) = h(x_1 - x_2, y_1 - y_2)$ the output function as a convolution of the input and the impulse response function:

$$g_2(x_2, y_2) = \iint h(x_2 - x_1; y_2 - y_1)\, g_1(x_1, y_1)\, dx_1\, dy_1$$

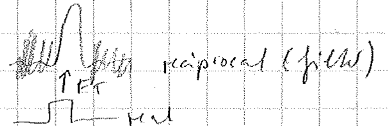
FT: convolution → product in reciprocal space

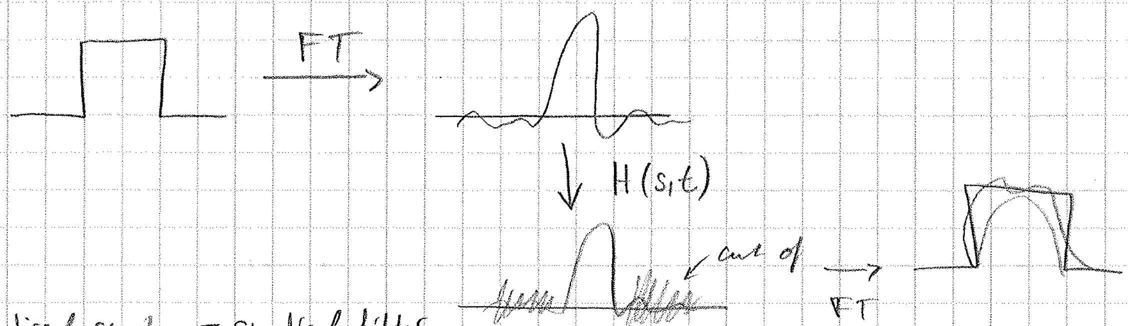
The property of linear systems leads to the fact that often instead of real space coordinates, the Fourier transformed of the parameters in the system are used, because of the convolution theorem:

reciprocal coord.

$$G_2(s,t) = H(s,t) \cdot G(s,t)$$

G_2: measured; H: describes the optical system, filter (optical transfer function); G: input (unknown)





optical system = spatial filter

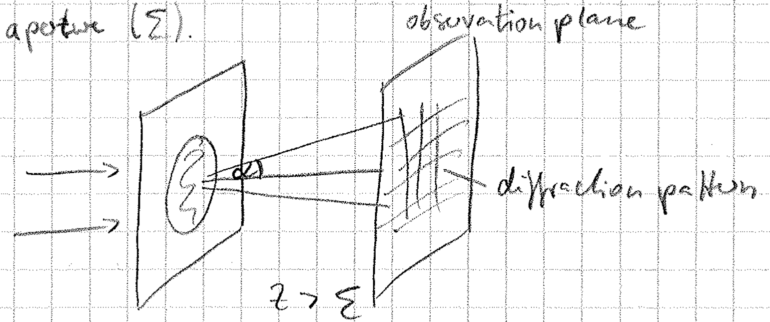
problem
$h(x_1-x_0, y_1-y_0, z)$ still contains $\frac{\exp[ikr_{01}]}{r_{01}}$ — $\sqrt{(x-x_1)^2+\ldots}$ we can not handle
→ try to derive simplification

03.11.2016

$h(x_1-x_0, y_1-y_0, z)$ still contains $\frac{\exp[ikr_{01}]}{r_{01}}$

⇒ <u>Fresnel- and Fraunhofer diffraction</u>

These approaches assume that the distance z of the aperture plane is large compared to the observation plane and large compared to the diffractive aperture (Σ).



Furthermore, also in the observation plane, the size of the diffraction pattern should be small compared to the distance z.
With these assumptions, we expand r_{01} in a series:

$$r_{01} = \sqrt{(x_1-x_0)^2+(y_1-y_0)^2+z^2} = z\sqrt{1+\frac{(x_1-x_0)^2+(y_1-y_0)^2}{z}}$$

$$= z \quad \frac{(x_1-x_0)^2+(y_1-y_0)^2}{2z} - \frac{\overset{\ll\pi}{\left[(x_1-x_0)^2+(y_1-y_0)^2\right]^2}}{8z^3}$$

Taylor exp.

$$\sqrt{1+a} = 1+\frac{a}{2}-\frac{a^2}{2\cdot 4}-\frac{a^3}{2\cdot 4\cdot 6}+\ldots$$

For angles $\leq 18°$, $\cos(\vec{n}, \vec{r}_{01}) = \cos\alpha = \frac{z}{r_{01}}$ equals 1 with an accuracy of 5%.
ignore for small angles

Therefore, $\cos\alpha = 1$ and $\frac{1}{r_{01}} \approx \frac{1}{z}$

$$U_{out} = \iint U e^{ikr_{01}} \cos(\ ) \, dx \, dy$$

Näherungen: Fraunhofer, Fresnel
which part of r_{01} contribute less to phase (cut off higher)
input
kann nicht vereinfacht werden, wegen Phase

how many λ goes in r_{01} → defines phase shift

$$e^{ikr_{01}} = \exp\left[i 2\pi \frac{r_{01}}{\lambda}\right]$$

The approach made for $\cos(\vec{n}, \vec{r}_{01})$ cannot be made for the exp.-factor, because $\lambda \ll r_{01}$, therefore, small changes of r_{01} (relative to z) cause relative large phase shifts.

As a result, we have to take into account at least the second term of the expansion. For the impulse-response-function we obtain:

$$h(x_1 - x_0, y_1 - y_0, z) = \frac{\exp(ikz)}{i\lambda z} \exp\left[\frac{ik}{2z}\left\{(x_1 - x_0)^2 + (y_1 - y_0)^2\right\}\right]$$

Under the assumption of Fresnel-diffraction the diffraction integral is:

fixed distance → no integral

$$\boxed{U(x_0, y_0, z) = \frac{\exp[ikz]}{i\lambda z} \iint_\Sigma U(x_1, y_1) \exp\left[\frac{ik}{2z}\left\{(x_1 - x_0)^2 + (y_1 - y_0)^2\right\}\right] dx_1 \, dy_1} \quad (1)$$

Fresnel transformation

Fresnel diffraction is valid if the contribution of the 3rd and higher terms can be neglected in the exponential factor.

We have to fulfill $\frac{2\pi}{\lambda} \cdot \frac{1}{8z^3} \left| (x_1 - x_0)^2 + (y_1 - y_0)^2 \right|^2_{max} \ll \pi$

We get for the distance $z^3 \ggg \frac{1}{4\lambda} \left| (x_1 - x_0)^2 + (y_1 - y_0)^2 \right|^2_{max}$ Condition for Fresnel

If we expand the exp.-factor of equation (1), we can rewrite equ. (1) as a Fourier transform:

can ignore because measure intensity $|U|^2$

$\ll \pi$ Fresnel, introduce lens to cancel → Fraunhofer

FT

$$(2) \quad U(x_0, y_0, z) = \frac{\exp[ikz]}{i\lambda z} \exp\left[\frac{ik(x_0^2 + y_0^2)}{2z}\right] \iint_\Sigma U(x_1, y_1) \exp\left[\frac{ik}{2z}\left\{(x_1^2 + y_1^2)\right\}\right] \exp\left\{\frac{2\pi i}{\lambda z}(x_0 x_1 + y_0 y_1)\right\} dx_1 \, dy_1$$

$$f(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) \exp[-ikx] \, dx$$

The amplitude of the diffracted wave is obtained by a FT of the product $U(x_1, y_1) \exp\left[\frac{ik}{2z}(x_1^2 + y_1^2)\right]$.

try to get rid of this factor $\overset{!}{\approx} 1$

With further restriction, we obtain the case of Fraunhofer diffraction.

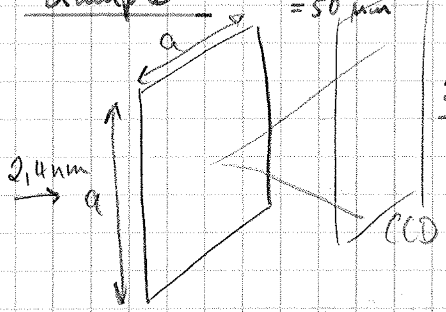
$$\left[\frac{ik}{2z}\left(x_1^2+y_1^2\right)\right] \ll \overset{\pi}{1}$$

$$\frac{2\pi}{\lambda}\frac{1}{2z}\left(x_1^2+y_1^2\right) \ll \pi$$

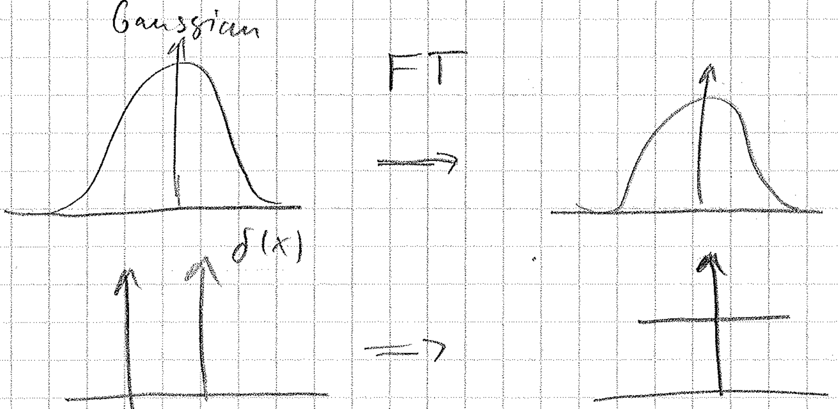
$$\Rightarrow \quad z \gg \frac{1}{\lambda}\left|x_1^2+y_1^2\right|_{max}$$

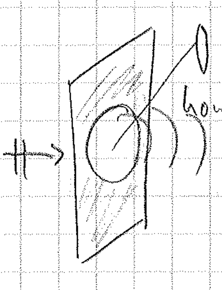
<u>Fraunhofer far field condition</u> $\boxed{\rightarrow \text{obtain 2D FT}}$!

<u>Example:</u> $a = 0.05\,mm = 50\,\mu m$ $\quad \lambda = 2{,}4\,nm$ soft x-ray



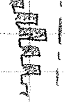
$$\frac{2a^2}{\lambda} = 2{,}08\,m$$ distance to fulfill Fraunhofer condition





introduce e.g. a lens or Fresnel zone plate

how to solve this?



08.12. : Uso-Meeting in Bessy II — 07.11.2016

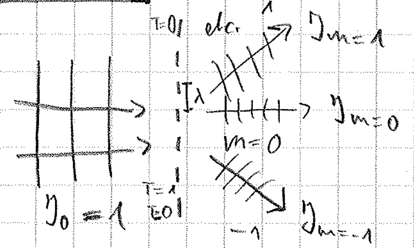
→ no lecture, but walk around HZB

$15^{\underline{00}}$ Einsteinstr. 15 am Eingang

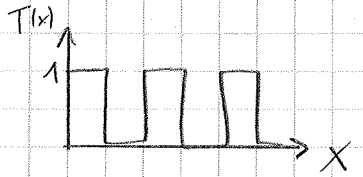
21.11. : Schneider schickt Vertretung : Superresolution ~~Spectroscopy~~ Microscopy

12.12. : letzte VL bei Schneider

Exercise



$$\eta_m = \frac{I_m}{I_0} \qquad \left(\sum \eta_m = 1 \quad \text{any phase shift}\right)$$



$$T(x) = \frac{1}{2} + \frac{2}{\pi} \sum_{m=1,3,5\ldots}^{\infty} \frac{1}{m} \sin(m\sigma x) \qquad \sigma = \frac{1}{\Lambda}$$

$$\sin(x) = \frac{\exp[ix] - \exp[-ix]}{2i}$$

Fraunhofer — do it in 1D

measure intensity

$$u(x_0, z) = \frac{\cancel{\exp(ikz)}}{\cancel{i\lambda z}} \, \cancel{\frac{ik x_0^2}{2z}} \int u(x_1) \exp\left[-ik \frac{x_0 x_1}{z}\right] dx_1$$

$$\propto \int u(x_1) \exp\left[-ik \frac{x_0 x_1}{z}\right] dx_1$$

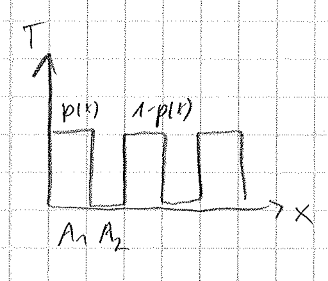
$\uparrow$ $T(x_1)$ Grating

exp → δ functions with different Prefactor

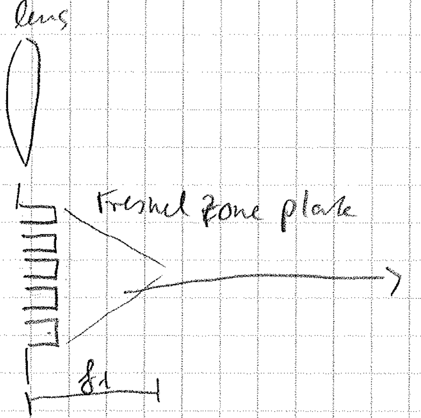
$$\eta_0 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

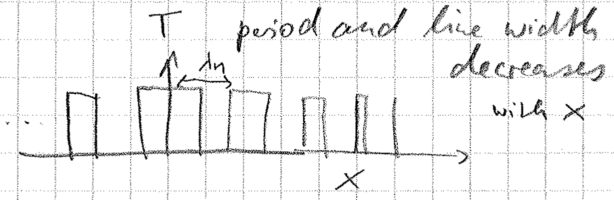
+ or − order symmetric

$$\eta_{m=1} = \frac{1}{\pi^2} \quad \Rightarrow \quad \eta_m = \frac{1}{m^2 \pi^2}$$



Optical Elements





$$T(x,y) = \frac{1}{2} + \frac{2}{\pi} \sum_{m=1,3,5\dots}^{\infty} \frac{1}{m} \sin\left[m \frac{\pi}{\lambda f} (x^2+y^2)\right] \qquad r^2 = x^2+y^2$$

Fresnel diffraction

$$\exp\left[\pm \frac{im\pi}{\lambda f}(x_1^2+y_1^2)\right]$$

$$U(x_1,y_1)$$

$$U(x_0,y_0,z) = \frac{\exp[-ikz]}{i\lambda z} \underset{\Sigma}{\iint} \frac{1}{2} + \frac{2}{\pi} \sum \frac{1}{m} \sin\left[\frac{m\pi}{\lambda f}(x_1^2+y_1^2)\right] \cdot \exp\left[\frac{ik}{2z}(x_1^2+y_1^2)\right]$$

$$\cdot \exp\left[\frac{2\pi i}{\lambda z}(x_0 x_1 + y_0 y_1)\right] dx_1\, dy_1$$

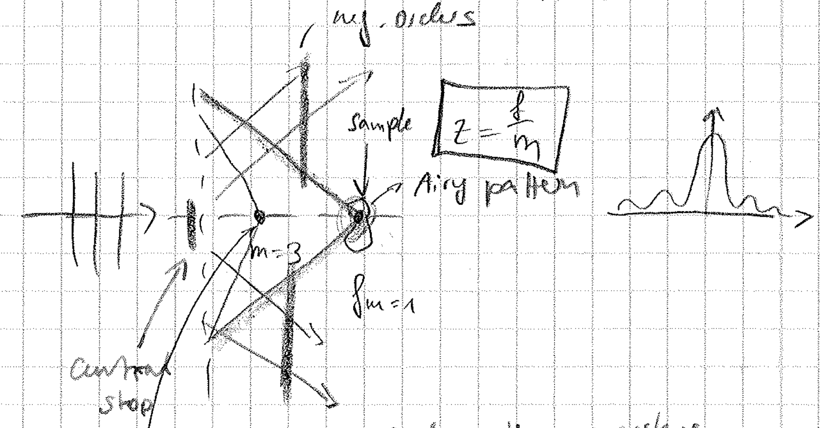
ignore $\frac{1}{2} \to 0$ Order

$$\frac{ik}{2z}(x_1^2+y_1^2) = \frac{im\pi}{\lambda f}(x_1^2+y_1^2) \Rightarrow z = \frac{\pm f}{m}$$

FT of an aperture in a distance f/m

$| \xrightarrow{FT} \text{sinc}(x)$

$$\eta_m = \frac{1}{\pi^2 m^2}$$



→ OSA : Order sorting aperture

→ select one order

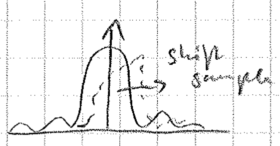
→ raster scan microscop spacial resolved

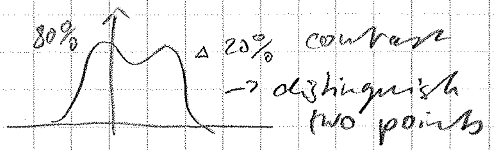
aperture 3x larger than m=1 and point 3x smaller

BUT less intensity

$$\eta_m = \frac{1}{m^2\pi^2}$$

Rayleigh resolution?





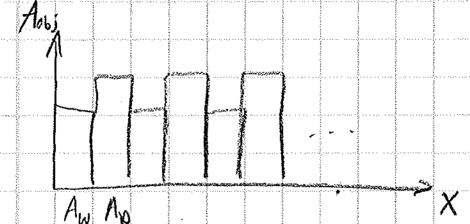
depends on $\frac{\lambda}{NA}$ $\sim \sin\alpha$

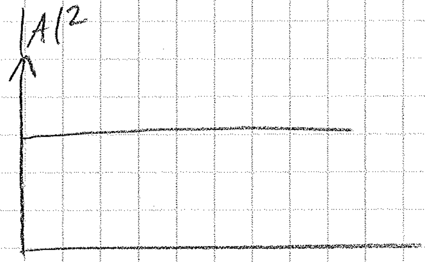
10th Nov. 2016

$\sim e^{-\tau t} \xrightarrow{FT} \nu$ $\qquad E = h\nu$

(τ für Hohlräume größer als für Atome → kleinere Energie (Hohlräume))
low frequencies ~ higher energies

$\delta = 0{,}61 \frac{\lambda}{NA}$ Rayleigh resolution





$A = A_0 e^{-ik\tilde{n}t}$

$\tilde{n} = 1 - \delta - i\beta$ refractive index

absorption

phase shift

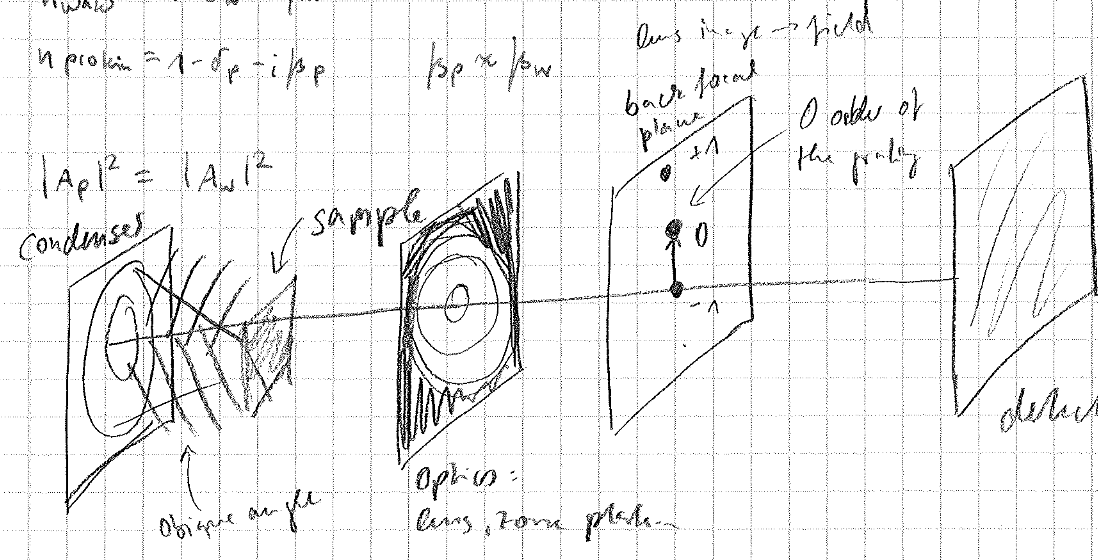
$A = A_0 \exp(-ik(1-\delta-i\beta)t)$ $\qquad |A|^2 = e^{\beta \cdot\cdot}\, e^{-\cdot\cdot\delta}$

~ verschwindet

$n_{water} = 1 - \delta_w - i\beta_w$

$n_{protein} = 1 - \delta_p - i\beta_p$ $\qquad \beta_p \approx \beta_w$

$|A_p|^2 = |A_w|^2$



→ microscope
(→ fig. 1 Schneider paper)

Fresnel (eq. 4)
include lens (eq. 5)
define aperture (eq. 6)
Fresnel propagation

lens $m=1$ (1 lens)

lens : plane 2 : $\sim \exp\left[\frac{i m \pi}{\lambda f}(x^2+y^2)\right]$

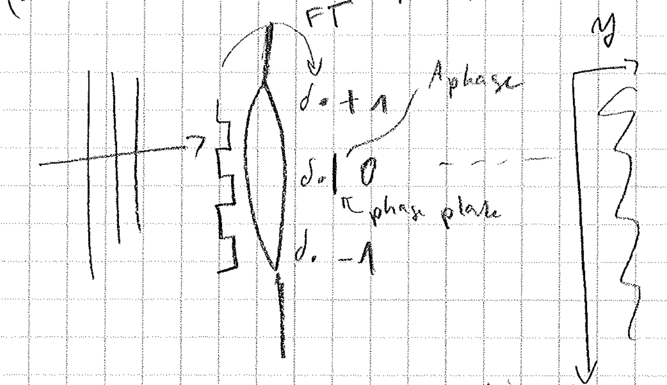
to fulfill lens law: move planes $\rightarrow \exp = 1$

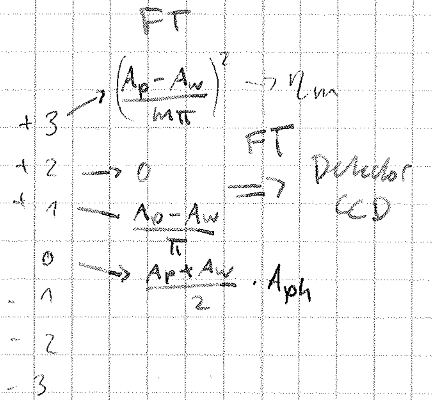
$\rightarrow$ 2 2D FT 1. FT in Fourierplane (back focal plane)

Fourierplane $\rightarrow$ image plane corresponds to 2nd FT $\Rightarrow$ image

aperture lens : defines how many orders we can see on back focal plane
↓
spatial filter

(cancel 1st order $\rightarrow$ dark field) $|A_{Det}|^2 = I_{image}$





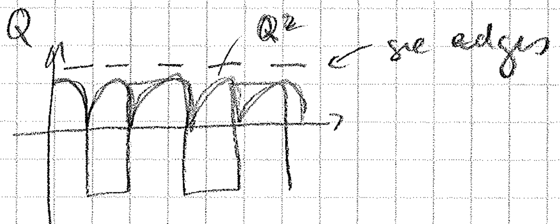
$$A_{obj} = \frac{A_p + A_w}{2} + \frac{2(A_p - A_w)}{A} \sum_{m=1,3,5}^{\infty} \frac{1}{m} \sin(mGx)$$

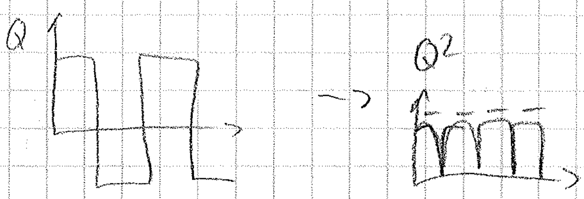
(15) δ-Fct. are relatively shifted to each other

(19), (20) spatial filtering $\rightarrow$ limited aperture

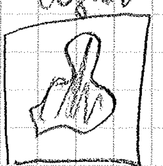
FT back to image plane (26)

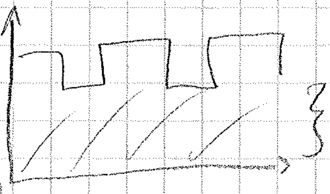
$n_{ph} = 1 - \delta_{ph} - i\beta_{ph}$





Object





$\boxed{\text{dark field imaging!}}$

Phase plate : switch from phase to absorption contrast

$A_{ph} \rightarrow \infty$ dark field imaging

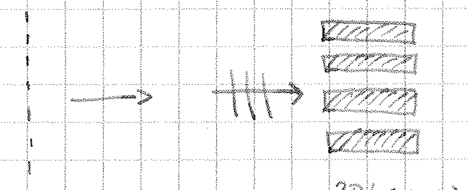
Absorption contrast : S_3 max $\Rightarrow \cos(..) = 1 \Rightarrow S_4 = 0$

14. 11. 2016

Volume Diffraction

now: no longer far field (→ no Fraunhofer, → no Fresnel)

so far:



planar structure: $\Delta z = 0$

2D (x, y)

3D (x, y, z)

$\Delta z \neq 0$

start with wave equation: $\nabla^2 E(x,z) + k_0^2\, \varepsilon(x,z)\, E(x,z) = 0$, $k_0 = \frac{2\pi}{\lambda}$

$\vec{E} = (E_x, E_y, E_z) \to E_y$

consider: plane wave polarized perpendicular to the plane of incidence ⇒ "H-mode"

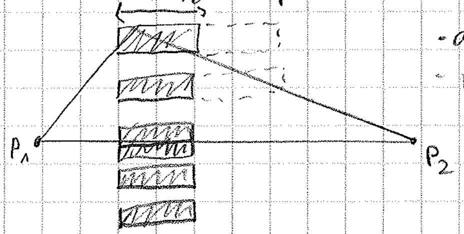
$\varepsilon(x,z)$ is the permittivity of the modulated region.

$\varepsilon = \tilde{n}^2 \qquad \tilde{n} = 1 - \delta - i\beta$

$\varepsilon(x,z) = \varepsilon_A\, p(x,z) + \varepsilon_B\, q(x,z) \qquad q(x,z) = 1 - p(x,z)$

$d/_{\Lambda/2}$ = aspect ratio

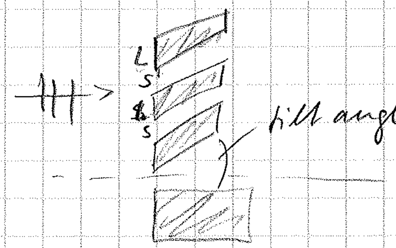
- absorption
- phase shift



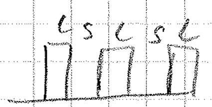
L - line

S - space

$L + S = \Lambda$ period



tilt angle φ



$$p(x,z) = \frac{L}{L+S} + \frac{2L}{L+S} \sum_{h=1,3,5}^{\infty} \mathrm{sinc}\left(h\pi \frac{L}{L+S}\right) \cos\left(h \vec{G}\vec{r}\right)$$

$$\vec{G} = \frac{2\pi}{\Lambda} \qquad \vec{G}\vec{r} = \frac{2\pi}{\Lambda}\left(x\cos\varphi - z\sin\varphi\right)$$

$$\varepsilon(x,z) = \bar{\varepsilon} - \Delta\varepsilon \frac{2L}{L+S} \sum_{h=1,3,5}^{\infty} \mathrm{sinc}\left(h\pi \frac{L}{L+S}\right) \cos\left(h\vec{G}\vec{r}\right)$$

$$\bar{\varepsilon} = \varepsilon_B + (\varepsilon_A - \varepsilon_B)\frac{L}{L+S} = \tilde{n}_B^2 + (\tilde{n}_A^2 - \tilde{n}_B^2)\frac{L}{L+S} \qquad \Delta\varepsilon = \varepsilon_A - \varepsilon_B$$

next step: put ε into wave equation superpos. of plane waves

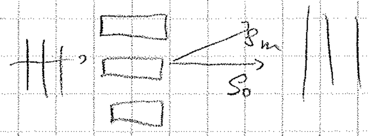
Ansatz: $E(x,z) = \sum_{m=-\infty}^{\infty} E_m(x,z) = E_0 \sum_{m=-\infty}^{\infty} e^{-i(\vec{S}\cdot\vec{r})} \cdot A_m(z)$ attenuation

no attenuation of wave field

$\vec{S}_m \cdot \vec{r} = S_{m,x} \cdot x + S_{m,z} \cdot z$

$\vec{S}_m = \vec{S}_0 + m\vec{G}$ ← grating vector $\quad m = 0, \pm 1, \dots$

(transfer energy between orders)



(similar to Laue condition) → which peaks do I get?

→ constr. interference

$S_{m,x} = k \sin\theta_{in} + m G \cos\varphi$

$S_{m,z} = k \cos\theta_{in} - m G \sin\varphi \qquad k = \frac{2\pi\sqrt{\varepsilon}}{\lambda}$

put in wave equ.: (results)

$$\sum_{m=-\infty}^{\infty} e^{-i\vec{S}_m\vec{r}} \left\{ \frac{d^2A(z)}{dz^2} - 2iS_{m,z}\frac{dA(z)}{dz} - (S_{m,x}^2 + S_{m,z}^2) A_m(z) \right.$$

$$\left. + k_0 \bar{\varepsilon} A_m(z) + k_0^2 \Delta\varepsilon\, A_m(z) \frac{2}{c+s} \sum \mathrm{sinc}\left(h\pi \frac{c}{c+s}\right) \cos(hG\vec{r}) \right\} = 0$$

$$\cos(h\vec{G}\vec{r}) = \frac{\exp(ih\vec{G}\vec{r}) + \exp(-ih\vec{G}\vec{r})}{2}$$

(grating) h Fouriercoefficient for grating

$$e^{-i\vec{S}_m\vec{r}} e^{\pm ih\vec{G}\vec{r}} = e^{-i(\vec{S}_m \mp h\vec{G})\vec{r}} = e^{-i\vec{S}_{m\mp h}\cdot\vec{r}}$$

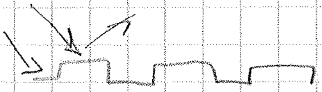
$$\vec{S}_{m\mp h} = \vec{S}_0 + m\vec{G} \mp h\vec{G} = \vec{S}_0 + (m \mp h)\vec{G}$$

metal: (reflection) $\frac{dA(z)}{dz}$ slowly varying → neglect

$\frac{d^2A(z)}{dz}$ cannot be neglected (→ reflection) response for

↑ include forward and backward waves

17. 11. 2016
(Volume gratings)

use volume gratings 

(reflection: e.g. for Raman spectroscopy)
(s. andere VL)

$$|\vec{s}_m|^2 = s_{m,x}^2 + s_{m,z}^2$$

$$\sum_{-\infty}^{\infty} \exp(-i\,\vec{s}_m\vec{r})\left\{\frac{d^2A_m(z)}{dz^2}\; 2i s_{m,z}\,\frac{dA(z)}{dz} - \left(|s_m|^2 - k_0^2\bar{\varepsilon}\right)A_m(z) + k_0^2\Delta\varepsilon A_m(z)\frac{L}{L+S}\right.$$

$$\left.\cdot\sum_{h=1,2,3\ldots}^{\infty}\operatorname{sinc}\left(h\pi\frac{L}{L+S}\right)\left[\exp\left(ih\vec{G}\vec{r}\right) + \exp\left(-ih\vec{G}\vec{r}\right)\right]\right\} = 0$$

$$\exp(-\vec{s}_m\vec{r})\cdot\exp(\pm ih\vec{G}\vec{r}) = \exp(-i(\vec{s}_m \mp h\vec{G})\vec{r}) = \exp(-i\vec{s}_{m\mp h}\cdot\vec{r})$$

$$\vec{s}_{m\mp h} = \vec{s}_0 + m\vec{G} \mp h\vec{G} = s_0 + (m\mp h)\vec{G}$$

$$\sum_{m=-\infty}^{\infty}\exp(-i\vec{s}_m\vec{r})\left\{\frac{d^2A(z)}{dz^2} - 2is_{m,z}\frac{dA_m(z)}{dz} - \left(|s_m|^2 - k_0^2\bar{\varepsilon}\right)A_m(z)\right\}$$

$$+k_0^2\Delta\varepsilon\frac{L}{L+S}\sum_{m=-\infty}^{\infty}\sum_{h}^{\infty}\operatorname{sinc}\left(h\pi\frac{L}{L+S}\right)\cdot A_m(z)\left[\exp(-i\vec{s}_{m-h}\vec{r}) + \exp(-i s_{m+h}\vec{r})\right] = 0$$

$$\left[\sum_{m=-\infty}^{\infty}\sum_{h=1,2,3}^{\infty}A_m B_{m-h} = \sum_{m=-\infty}^{\infty}\sum_{h=1,2,3}^{\infty}A_{m+h}B_m\right]$$

$$\sum_{m=-\infty}^{\infty}\cancel{\exp(-i\vec{s}_m\vec{r})}\left\{\overset{0}{\cancel{\frac{d^2A(z)}{dz^2}}} - 2is_{m,z}\frac{dA_m(z)}{dz} - \left(|s_m|^2 - k_0^2\bar{\varepsilon}\right)A_m(z) + k_0^2\Delta\varepsilon\frac{L}{L+S}\right.$$

is $\{\ \}_m = 0$

$$\left.\cdot\sum_{h=1,2,3}^{\infty}\operatorname{sinc}\left(h\pi\frac{L}{L+S}\right)\cdot\left[A_{m+h}(z) + A_{m-h}(z)\right]\right\} = 0$$

$\cancel{\frac{d^2A(z)}{dz^2}}$

neglect reflection
transm. case $\frac{d^2A}{dz^2} = 0$
(slowly varying)

$$\frac{d^2A(z)}{dz} + \frac{dA(z)}{dz} + A(z) = 0 \;\Rightarrow\; A(z) = \exp(\ldots z)$$

$$c^2 + c + 1 = 0 \quad \rightarrow \exp(\pm cz)$$

$\pm s_z$
(travel direction / propagating direction)

grating matrix

$$\begin{bmatrix}\vdots\\ \frac{dA_1(z)}{dz}\\ \frac{dA_0(z)}{dz}\\ \frac{dA_{-1}(z)}{dz}\\ \vdots\end{bmatrix} = \begin{bmatrix}\ddots & & & \\ a_0 & \ddots & & \\ & a_1 & & \\ b & & a_0 & \\ & & & a_{-1} \\ & & & & \ddots\end{bmatrix}\begin{bmatrix}A_1(z)\\ A_1(z)\\ A_0(z)\\ A_{-1}(z)\end{bmatrix}$$

$m = h = 30$
$2h+1 = 61\times 61$
2 complex matrix

opt. const. and grating

$$a_m = \frac{i|s_m|^2 - k_0^2\bar{\varepsilon}}{2s_{m,z}}, \quad b_{mh} = \frac{k_0^2\Delta\varepsilon L}{2is_{m,z}(L+S)}$$

$$\frac{d\underline{A}(z)}{dz} = \underline{\underline{M}}\,\underline{A}(z)$$

eigenvalues

$$A_m(z) = \sum_n q_{mn}\left[c_n \exp(\chi_n z)\right]$$

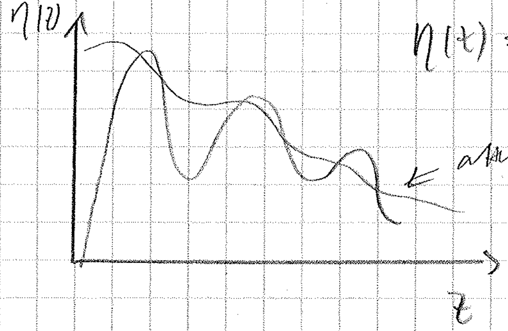
$$\begin{bmatrix} \vdots \\ A_1(z) \\ A_0(z) \\ A_{-1}(z) \\ \vdots \end{bmatrix} = \begin{bmatrix} q & & \\ & q_{01} & \\ & & \ddots \end{bmatrix} \begin{bmatrix} c_n e^{(\chi_n z)} \\ \vdots \\ c_0 e^{(\chi_0 z)} \\ \vdots \end{bmatrix}$$

$$\Sigma\,\eta_m(z) = 1$$

$$\shortparallel$$

$$\text{starting cond.}\quad \begin{bmatrix} \vdots \\ 0{,}0 \\ 1{,}0 \\ 0{,}0 \\ \vdots \end{bmatrix}$$

$$\eta(z) = |A_m(z)|^2$$



← attenuation Pendellösung

(s. Skript)

21.11.2016

Fluorescence, deconvolution and superresolution microscopy
(→ slides)

fluorescence: absorption of a photon with emission of longer wavelength photon

image: convolution of image and point spread function (of microscope, wavelength, diffraction...)

$$\text{image} = \text{object} \circledast \text{psf}$$

Superresolution: breaking the diffraction limit of ~170 nm in light microscopy

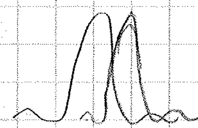
→ trick: fluorescence

· fluorescence microscope with electron microscope resolution?

· methods: PALM/STORM, STED, structured illumination

· PALM/STORM: photoactivate or photoconvert dyes

(dark state → fluorescencing state)



single molecules can be localized precisely

two close molecules: cannot distinguish

STED: an alternative to random, infrequent activation of fluorescence states is PSF "engineering"

Structured illumination: Moire fringes

→ convert higher frequencies to lower frequencies (detectable, "visible")

rotate / different angles

24.11.2016

-> book chapter: Volume gratings (G. Schneider, S. Rehbein, and S. Werner)

photonic crystal (as volume grating)
like electron in crystal? -> bloch function

- prefer material with less height zone -> Ni (Fig. 8.?)

- tilting structure: efficiency goes up

- Bragg diffraction (-> see higher orders Fig. 8.8) $\eta = \frac{1}{\pi m^2}$

- Why higher orders are interesting? -> larger aperture
larger resolution (but same structure size)

- processing: how to produce this structure? $m = 6$, $\eta = 0$ 0, ±1, ±3 ...

· lithography, ion beam etching (very precise)

=>

Photonic Crystals

· periodic dielectric structures, that can interact resonantly with radiation with wavelength comparable to the periodicity length of the dielectric lattice.

From Bragg Gratings to Photonic crystals in 5 steps.

· 1785: the first man-made diffraction grating was made by David Rittenhouse, who strung hairs between two finely threaded screws.

· 1913: Bragg formulation of X-ray diffraction by crystalline solids

· 1928: Bloch's Theorem describe the condition of electrons in crystalline solids (maths developed by Floquet in 1-D case already 1883!)

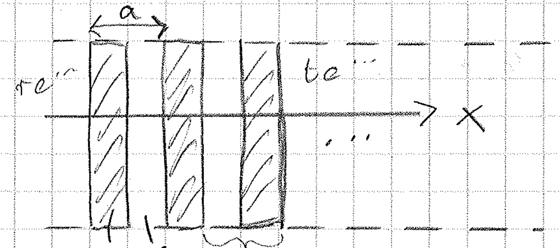
· 1976: A. Yariv and P. Yeh, study of dielectric multi layer stacks, waveguides and Bragg fibers

· 1987: Prediction of photonic crystals

1-D Photonic Crystals

=> a closer look

use: as mirror
monochromatic mirror ✓
(-> partly reflect and transmit light)
=> number of "mirrors" ↑ efficiency of selection

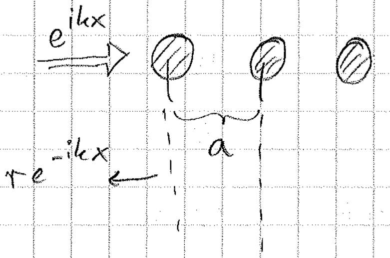


The dielectric constant is periodically modulated in one dimension

$\varepsilon(\vec{r}) = \varepsilon(x) = \varepsilon(x + na) \qquad n = 0, \pm 1, \pm 2 \ldots$

- to describe need 2nd order derivative: for- and backward propagating waves

Bragg scattering

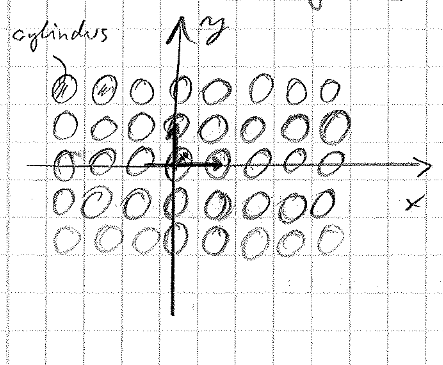


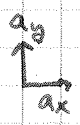
$$R = r \cdot e^{-ikx} + re^{-2ika} e^{-ikx} + re^{-4ika} e^{-ikx} + \ldots$$

$$\boxed{R = re^{-ikx} \frac{1}{1 - e^{-2ika}}}$$

mismatch affect bandgap (layers by large lithography, (sputtering))
drawback: 3D, have to be very accurate ; good: relative large structure
; good reflectivity - large refractive index difference (Tungsten)

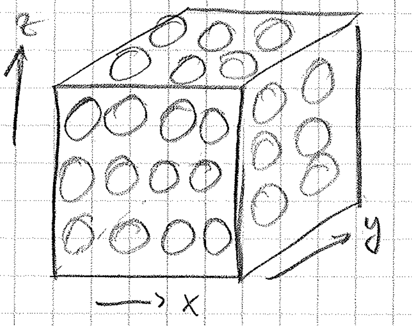
2-D Photonic Crystals





$\varepsilon(\vec{r}) = \varepsilon(x, y) = \varepsilon(x + na_x, y + ma_y)$
(2D-Bravais-lattice)
A cylinder is put at every lattice point.

3D photonic crystal



$$\varepsilon(\vec{r}) = \varepsilon(\vec{r} + n\vec{a}_1 + m\vec{a}_2 + l\vec{a}_3)$$

$$n, m, l = 0, \pm 1, \pm 2 \ldots$$

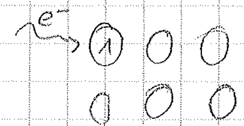
$$\frac{c^2}{\varepsilon(x)} \frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial t^2}$$

$$\hat{H}\Psi = E\Psi$$

28.11.2016

discrete energyvalues

$\Psi_H = \Psi_1 \ldots \Rightarrow$ solve SE to get energyeigenvalues



energybands in crystals

$\rightarrow$ Maxwell-Equ.

SE $\hat{H}\Psi = E\Psi$

Electrons (Bloch-wave function)	Photons in photonic crystals
As a result electrons can propagate in a periodic lattice without experience scattering with a proper dispersion relation (energy as a function of the wave vector)	Periodic modulation of the dielectric constant can affect the properties of photons in much the same way that ordinary semiconductor crystals affect the properties of electrons
$\Rightarrow$ band diagrams (electrons only scatter by impurities)	MW equ. in periodic Media $\hookrightarrow$ Master equ. photonic crystals

Maxwell's equations as an eigenvalue problem

$\Rightarrow$ Master equations for photonic crystalls (pc)

The calculation of the optical properties of pc starts with the source free Maxwell equations with electrical field $E(r,t)$, the dielectric displacement $D(r,t)$, the magnetizing field $\vec{H}(r,t)$ and the magnetic field $\vec{B}(r,t)$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

PC (linear medium)

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}(r,t) = \vec{D}(r,t)$$

$$\vec{H}(r,t) = \frac{1}{\mu_0} B(r,t)$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$\nabla \cdot \vec{D} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$\varepsilon(r) = \varepsilon \Sigma \ldots$ ← periodically

PC: Microscale periodic structure

Theoretical description: MW → Master equation → band gaps, defects..
(like SE for electronic systems)

start: (a) $\nabla \times E = -\frac{\partial B}{\partial t}$ and (b) $\nabla \times H = \frac{\partial D}{\partial t}$

(c) $H = \frac{1}{\mu_0} B$ and (d) $D = \varepsilon_0 \varepsilon_r E$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad | \nabla \times$$

$$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}(\nabla \times B) \quad | \nabla \times B = \nabla \times \mu_0 H = \mu_0 (\nabla \times H)$$

$$= -\frac{\partial}{\partial t} \mu_0 (\nabla \times H) \quad | (b)$$

$$= \frac{\partial}{\partial t} \mu_0 \frac{\partial D}{\partial t} \quad | D = \varepsilon_0 \varepsilon_r E$$

$$= \frac{\partial}{\partial t} \mu_0 \frac{\partial}{\partial t} \varepsilon_0 \varepsilon_r E \quad | \varepsilon_0 \mu_0 = \frac{1}{c^2}$$

$$\frac{1}{\varepsilon_r} \nabla \times (\nabla \times E) = -\frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} \qquad \nabla \times (\nabla \times A) = \nabla (\underbrace{\cancel{\nabla A}}_{\nabla E = 0 \text{ in free space}}) - \nabla^2 A$$

$$\frac{1}{\varepsilon_r} \nabla \times (\nabla \times E) + \frac{1}{c^2} \frac{\partial^2 E(r,t)}{\partial t^2} = 0$$

$$\nabla \times \left[\frac{1}{\varepsilon_r} \nabla \times H(r,t)\right] + \frac{1}{c^2} \frac{\partial^2 H(r,t)}{\partial t^2} = 0$$

With harmonic time dependency:

$$E(r,t) = E(r) e^{-i\omega t}$$

$$H(r,t) = H(r) e^{-i\omega t}$$

It results in an eigenvalue problem ⇒ Master equation of PC

$$\boxed{\begin{aligned} &\frac{1}{\varepsilon(r)} \nabla \times (\nabla \times E(r)) = \frac{\omega^2}{c^2} E(r) \\ &\nabla \times \left(\frac{1}{\varepsilon(r)} \nabla \times H(r)\right) = \frac{\omega^2}{c^2} H(r) \end{aligned}}$$

$\varepsilon(\vec{r} + a) = \varepsilon(\vec{r})$!!

Expand $\varepsilon(\vec r)^{-1}$ into a Fourier series:

reciprocal lattice vector

$$\varepsilon(\vec r)^{-1} = \sum_{G} \varkappa(\vec G)\, e^{i\vec G \vec r}$$

$\varkappa$ is hermitian $\varkappa(-\vec G) = \varkappa^*(\vec G)$

Analogous to the wavefunction for electrons in a crystall in solid state physics, we can apply the Bloch theorem to obtain periodic solutions of the Maxwell equations.

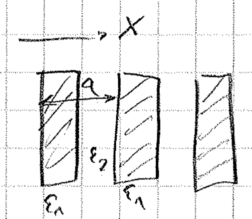
$E(\vec r)$ and $H(\vec r)$ are characterized by the wavevector k and the band index n:

1D

$$E_{kn}(r) = u_{kn}(r)\, e^{ikr}$$
$$H_{kn}(r) = v_{kn}(r)\, e^{ikr}$$

u_{kn}, v_{kn} are periodic with the grating

Periodic bandgaps in one dimension



1D PC

with $\varepsilon(x+a) = \varepsilon(x)$

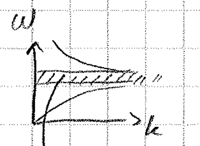
$$\varepsilon(r)^{-1} = \sum_{m=-\infty}^{\infty} \varkappa_m e^{i\frac{2\pi m}{a}x} \approx \varkappa_0 + \varkappa_1 e^{i\frac{2\pi}{a}x} + \varkappa_{-1} e^{-i\frac{2\pi}{a}x}$$

sinusoidal (not rectangular...)

solve: (forward and backward waves)

$$\frac{c^2}{\varepsilon(x)}\frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial t^2}$$

solutions for ω (energy) and k



certain energies do not propagate

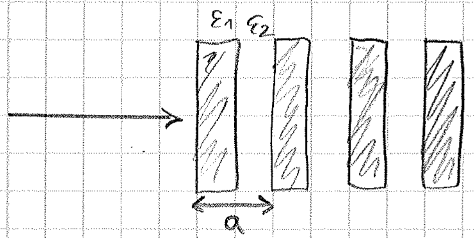
01.12.2016

	Quantum Mechanics	Electromagnetism
Field	$\Psi(\vec{r},t)=\Psi(r)e^{i\omega t}$	$H(\vec{r},t)=H(r)e^{i\omega t}$
Eigenvalue problem	$\hat{H}\Psi(r)=E\Psi$	$\Theta H(r)=\left(\frac{\omega^2}{c^2}\right)H(r)$
Operator	$\hat{H}=-\frac{\hbar^2\nabla^2}{2m}+V(r)$	$\Theta=\nabla\times\frac{1}{\varepsilon(r)}\nabla$

$$\left(\int\Psi\Psi^*\,d\tau \qquad |E|^2=I\right)$$

1D Photonic Crystal

$$\frac{c^2}{\varepsilon(x)}\frac{\partial^2E(x,t)}{\partial x^2}=\frac{\partial^2E(x,t)}{\partial t^2} \qquad (*)$$



spatial varying factor, periodic as well

$$\varepsilon^{-1}(x)=\sum_{n=-\infty}^{\infty}\underset{\uparrow\,\chi_n}{\chi_n}\,e^{i\frac{2\pi n}{a}x}\approx\underset{\uparrow\,\chi_0}{\chi_0}+\chi_1e^{i\frac{2\pi}{a}x}+\chi_1e^{-i\frac{2\pi}{a}x}$$

Apply Bloch theorem: $E(x,t)=u_k(x)e^{i(kx-\omega t)}=\sum E_m e^{i(k+\frac{2\pi m}{a})x-i\omega_k t}$

put function in $(*)$

Solve linear equation system (DGL system) with $u_k(x)$ periodic!

With the simplification $n=0,\pm1$

$$\chi_1\left[k+\frac{2(m-1)\pi}{a}\right]^2E_{m-1}+\chi_{-1}\left[k-\frac{(2m+1)\pi}{a}\right]^2E_{m+1}$$
$$\approx\left[\frac{\omega_k^2}{c^2}+\chi_0\left(k+\frac{2m\pi}{a}\right)^2\right]E_m$$

Want to know dispersion relation → which ω can propagate in the system $\omega(k)$

$$E_0\simeq\frac{c^2}{\omega_k^2-\chi_0c^2k^2}\left[\chi_1\left(k-\frac{2\pi}{a}\right)^2E_{-1}+\chi_{-1}\left(k+\frac{2\pi}{a}\right)E_{+1}\right]$$

$E_{-1}=\dots$, $E_{+1}=\dots$

$\vdots$

$$\Rightarrow\left(\omega_k^2-\chi_0c^2k^2\right)E_0-\chi_1c^2\left(k-\frac{2\pi}{a}\right)^2E_1=-\chi_{-1}c^2k^2E_0+\left[\omega_k-k_0c^2\left(k-\frac{2\pi}{a}\right)^2\right]\cdot E_{-1}=0$$

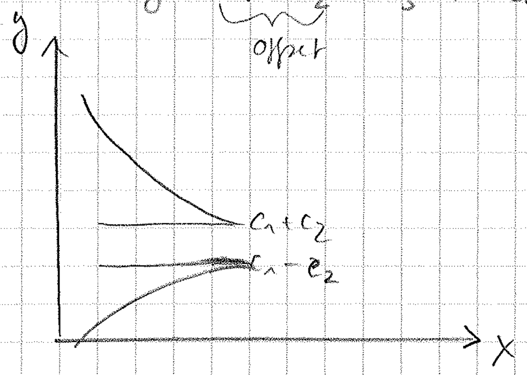
Determinante → non trivial solution

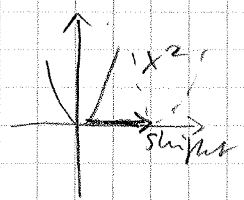
solution

$$\Rightarrow \boxed{\omega_\pm \approx \frac{\pi c}{a}\sqrt{\chi_0 \pm |\chi_1|} \pm \frac{ac}{\pi|\chi_1|}\left(\chi_0^2 - \frac{|\chi_1|^2}{2}\right)\left(k - \frac{\pi}{a}\right)^2}$$

grating parameters

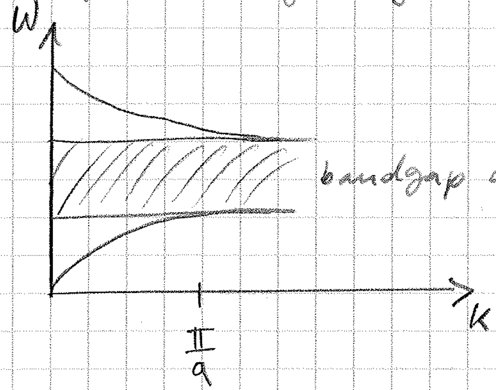
$$\rightsquigarrow y = \underbrace{c_1 \pm c_2}_{\text{offset}} \pm c_3\,(x - c_4)^2$$



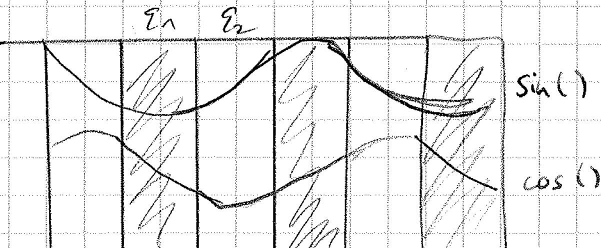


$\chi_1 = 0 \rightarrow$ no bandgap!

influence bandgap by ε

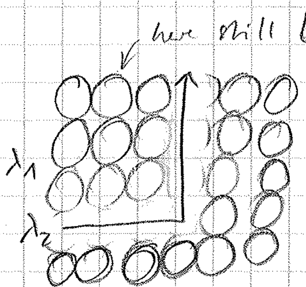


bandgap depending on the refractive index of material 1 and 2 (difference)



modes mix

for $k \approx \frac{\pi}{a}$ and $k \approx -\frac{\pi}{a}$



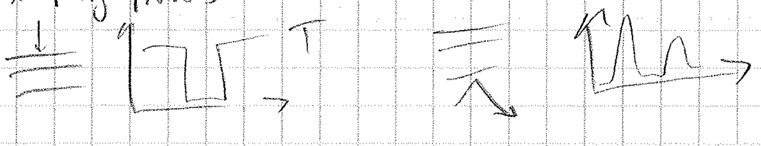
introduce dislocation

$\Rightarrow$ don't fulfill bandgap condition (no longer periodic system)

consider λ_2 is blogged in periodic crystal

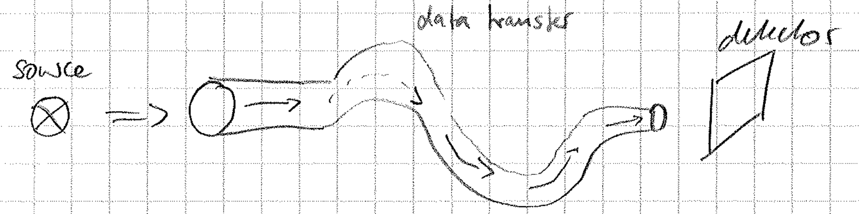
$\rightarrow$ now it can propagate

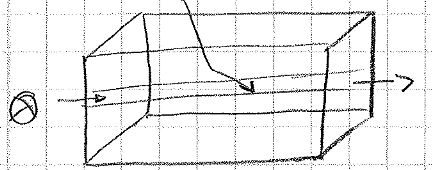
$\Rightarrow$ x-ray filters



05. 12. 2016

Wave guides



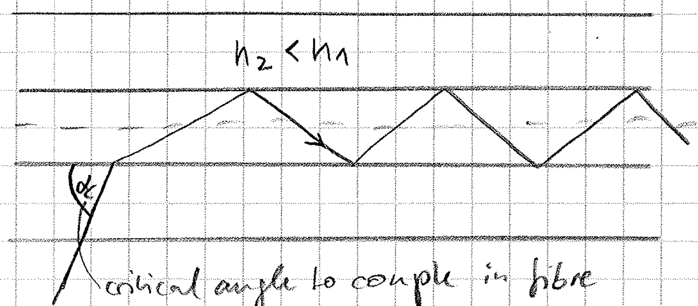


Wave guides are dielectric structures, which allow due to their refractive index distribution, that light propagates without loss over large distances.

Comparison of data transfer rate in glass fibres and Cu wires:

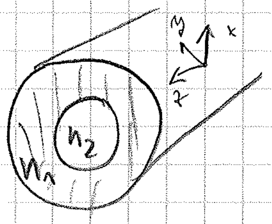
- in the Cu wire electrons move, the signal is transfered by momentum transfer of the electrons (with mass m_e)
- in the case of glass fibres, electromagnetic waves propagate in the material with refractive index n

$$c = \frac{c_0}{\sqrt{n}} \approx \frac{2}{3} c_0$$



critical angle to couple in fibre

$\alpha > \alpha_c$ no coupling in fibre



$\theta = \theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$

max incident angle

$n_0 \sin\alpha \sqrt{n_1^2 + n_2^2} = NA$

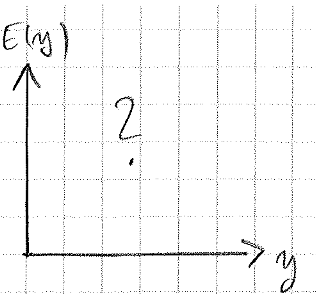
$n_0 = 1$

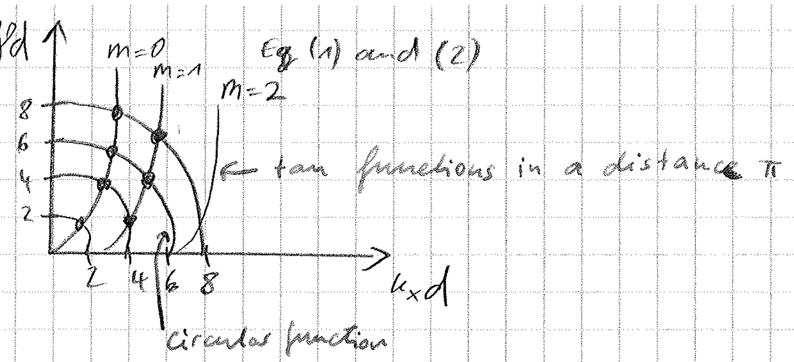
$n_1 = n_2 + 0.015$

$n_2 = 1.45$

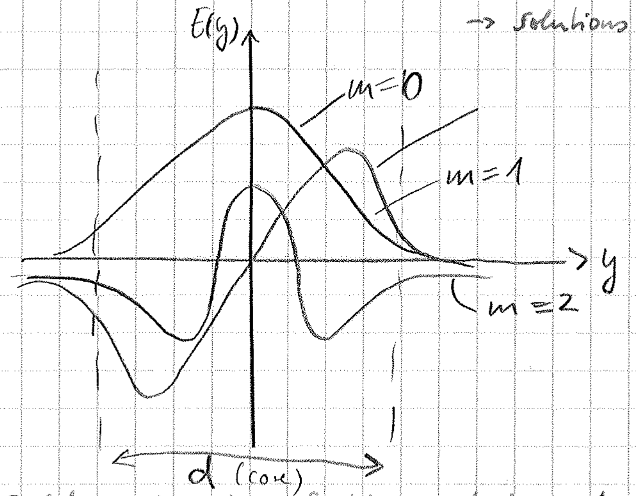
$\theta_c = 81.8°$

$\alpha_c = 12.1°$



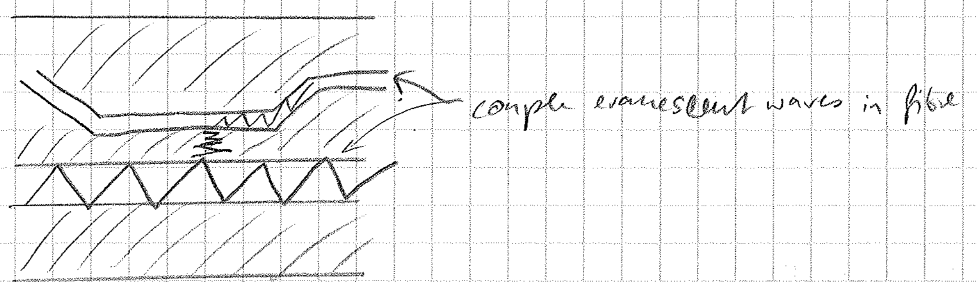


→ solutions: modes can propagate inside wave guide



Field distribution of the central mode $m=0$ and two higher modes $(m=1,2)$ in a planar wave guide!

modes prop. in wave guide → real solutions



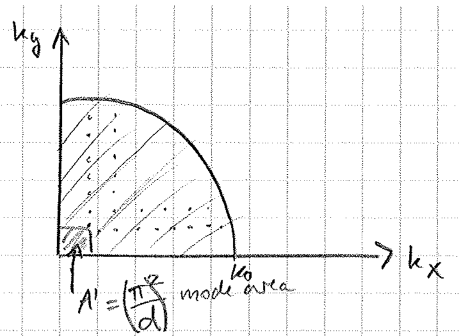
12. 12. 2016

<u>Boundmodes:</u> Real solutions of the eigenvalue problems. They represent waves, which are transmitted by total reflection in the fiber.

For a cylindric fibre: Number of modes depends on λ, d (diameter of fibre core) and the numerical aperture NA.

$$k_x^2 + k_y^2 + k_z^2 = k^2 \; ; \quad k_x^2 + k_y^2 \leq k^2 \qquad k_z \geq 0$$

$$A = \frac{\pi k_0^2}{4}$$



$\Rightarrow$ number of modes $= \frac{A}{\text{mode area}}$

$2k_x d = 2\pi m_x$; $m_x = 1,2,3\ldots$

$2k_y d = 2\pi m_y$; $m_y = 1,2,3\ldots$

$N = \frac{1}{2}\nu^2 \Rightarrow N = \frac{1}{2}\frac{\pi^2 d^2}{\lambda^2} NA^2$?

$n = \frac{2d}{\lambda} NA$ mode propagate

$n^2 = k_0^2\, NA^2 = k_0^2 n^2 m^2 \theta_c$

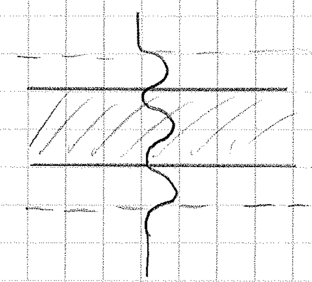
$\frac{A}{A'} = \frac{\pi d^2}{\lambda^2}$

normalised frequency in the fiber: $\nu = \frac{\pi d}{\lambda} NA$

Example: $\left.\begin{aligned} NA &= 0.2 \\ d &= 600\,\mu m \\ \lambda &= 0{,}308\,\mu m \end{aligned}\right\} \Rightarrow N = 750.000$ modes

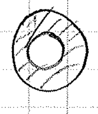
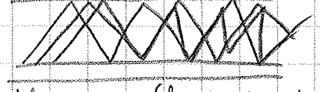
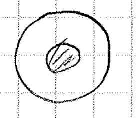
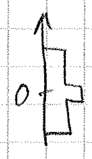
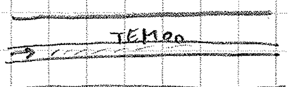
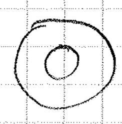
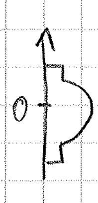
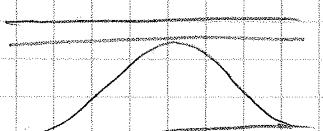
unbound modes

Complex solutions of the eigenvalue equation, which loos energy by refraction and reflection.



large amplitude outside the core.

Waveguide types:

	cross section	refractive index	beam propagation
multimode glass fibre			 telecom. (for communication)
single mode			 laser (telecom. ?) (for lasers)
multimode gradient fiber			

Fiber attenuation mechanism:

1. material absorption
2. scattering loss
3. bending loss
4. radiation loss (due to mode coupling)
5. leaky modes

- intrinsic loss
  - → intrinsic loss by absorption
    - → electronic transition problem: UV light
    - → vibrational losses
  - → intrinsic loss by scattering
    - → Rayleigh scattering
    - → Raman scattering

- extrinsic loss → absorption loss by impurities
  - → transition metal absorption
  - → absorption by the OH-vibration (OH in glass)
  - → loss by scattering caused by irregular structures → interface → pores, bending of the fibre etc.

loss mechanism in glass fibers:

Beer-Lambert law

$$I = I_0 \, e^{-\alpha L}$$

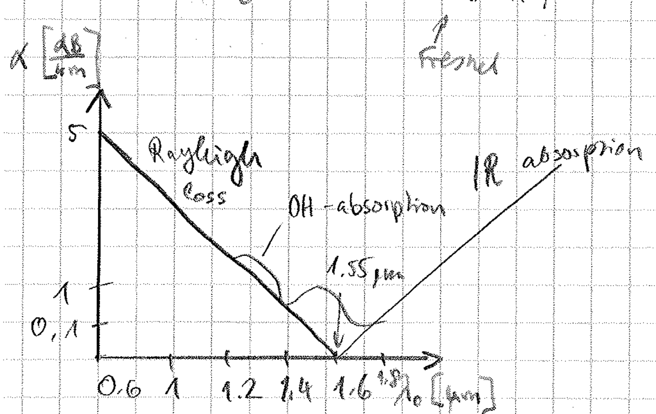
L: length of the fibre

α: absorption of coefficient

including reflection loss at the end of the fiber:

$$\eta = \frac{I}{I_0} = 1 - 2\left(\frac{n-1}{n_{air}+1}\right)^2 e^{-\alpha L}$$

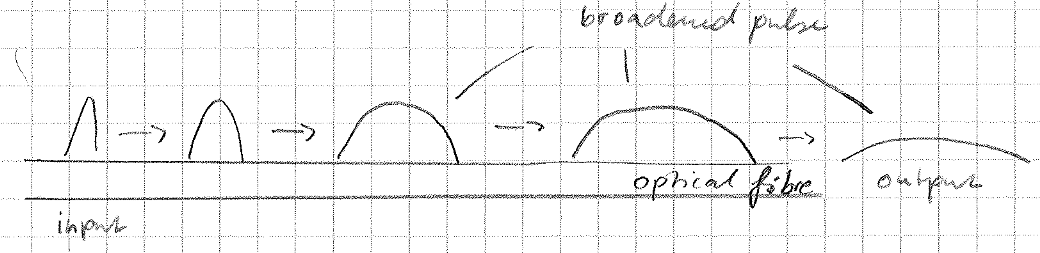
$\left(\frac{n-1}{n_{air}+1}\right)^2$: Fresnel

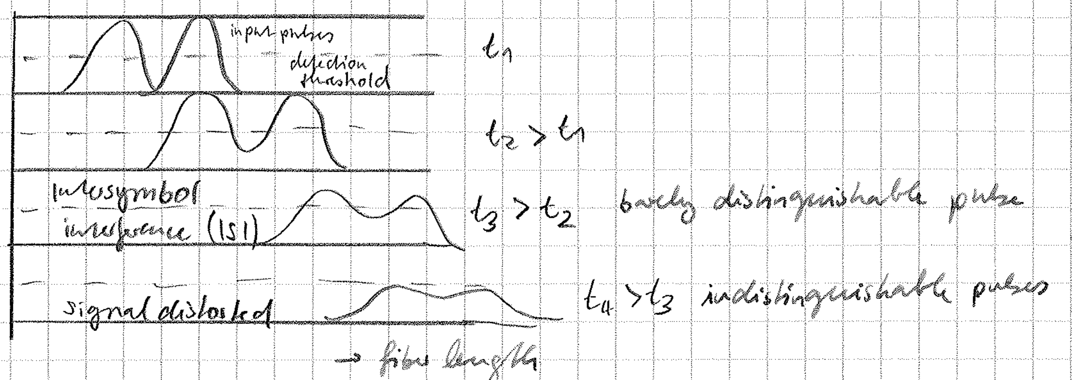


Conclusion:

Rayleigh scattering is dominating in the UV range while multi photon absorption limits the usable wavelength range in the IR region (~1.55 µm).

Fibre dispersion results in optical pulse broadening and hence signal degradation





for $\lambda = 1550$ nm

$f = 2 \cdot 10^{14}$ Hz $\hat{=}$ 200 000 $\frac{Gbit}{s}$ ⇒ $2 \cdot 10^9$ phone calls at the same time (one phone call 20 $\frac{kBit}{s}$)

Cu-wire: max 10^{11} Hz $\hat{=}$ 100 Gbit/s

Glass fibre: 1.3 - 1.6 µm : less than 0.3 $\frac{dB}{km}$ damping is possible

frequency band permits 43 Tbit/s

In practice: 14 $\frac{TBit}{s}$ (2006)

Advantages and disadvantages

⊕	⊖
cheap	electrical energy (⇒ multi plexer)
low damping	cabeling/wiring/connection of fibre difficult
large bandwidth	expensive components
safety	