Fundamentals of Optical Sciences

WS 2015/2016 **2 . Exercise** **26.10.2015**

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Prepare your answers for the exercise on 02.11.2015.

Exercise 1

Refraction at a spherical boundary

a) Derive the ABCD matrix for a refractive spherical boundary

b) Find the determinant of the matrix

Focussing a beam with a sphere

c) Use the spherical boundary matrix to find the focal length for glass bead. At what distance f is a ray parallel to the optical axis at $y = 0.8$ mm focussed behind a glass bead with diameter $d = 2$ mm?

Exercise 2

A Gaussian beam with $w_0 = 1$ cm is focused by a thin lens of focal length $f = 2$ cm. The lens is placed at the focus of the original Gaussian beam. Assume an optical wavelength of $\lambda = 1.0$ μm.

a) At what distance from the lens does the new focus occur?

b) What is the spot size at the new focus? Give numbers for both an ideal and a realistic lens.

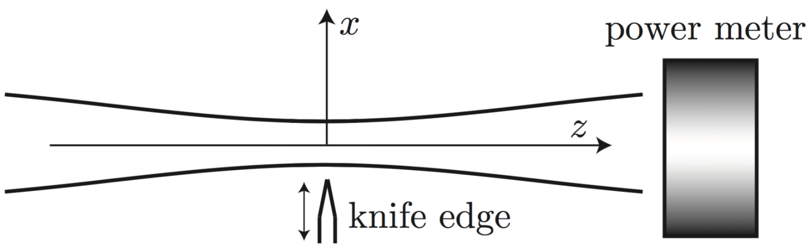
c) What is the far-field expansion angle?

Exercise 3

A common technique in the laboratory for measuring the beam waist parameter of a Gaussian beam is illustrated in the diagram. A Gaussian beam is incident on an optical power meter, which registers the total power of the incident beam. A knife edge can be translated in the transverse direction to block part of the beam (i.e., if the position of the knife edge is x_{knife}, then the part of the beam in the region $x < x_{\text{knife}}$ is blocked from reaching the power meter). The "10-90" rule is to measure the knife edge position $x_{10\%}$ where the power meter reads 10% of the total beam power, and then the position $x_{90\%}$ where the power meter reads 90% of the total beam power. Then the beam radius $w(z)$ at the knife-edge location z along the beam is given by

$$w(z) = \alpha \, |x_{10\%} - x_{90\%}| \tag{1}$$

where α is a constant factor. Calculate the numerical value of α.



Exercise 4

It is possible to derive a formal solution for the electric and magnetic fields. Using the so-called retarded potentials, show (in the vaccum)

$$\text{a)} \quad \vec{E}(\vec{r},t) = \frac{1}{4\pi\epsilon_0} \int \left(\frac{\rho(\vec{r}\,',t_{\text{ret}})}{s^2}\hat{e}_s + \frac{\dot{\rho}(\vec{r}\,',t_{\text{ret}})}{c\,s}\hat{e}_s - \frac{\dot{\vec{j}}(\vec{r}\,',t_{\text{ret}})}{c^2\,s} \right) \mathrm{d}V\,'$$

$$\text{b)} \quad \vec{B}(\vec{r},t) = \frac{\mu_0}{4\pi} \int \left(\frac{\vec{j}(\vec{r}\,',t_{\text{ret}})}{s^2} + \frac{\dot{\vec{j}}(\vec{r}\,',t_{\text{ret}})}{c\,s} \right) \times \hat{e}_s \, \mathrm{d}V\,'$$

where $\vec{s} = \vec{r} - \vec{r}\,'$ and $\hat{e}_s = \vec{s}/s$.

Exercise 5

An elegant way to describe how optical devices modify the polarization of light beams is to use Jones matrices.

a) Consider a beam of linearly polarized light passing subsequently through two ideal linear polarizers. A polarizer is an optical filter that passes light with a component along its transmission axis. The angle between the polarizers is 30°. The Jones matrix of a linear polarizer with an angle of α with respect to the x axis is

$$M_{\mathrm{LP}}^{(\mathrm{J})} = \begin{pmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{pmatrix}.$$

The angle between the light polarization and the transmission axis of the first polarizer is 15°. Calculate the Jones vector after passing both polarizers.

b) Now consider an arrangement where the second polarizer is exchanged with a quarter-wave plate. A quarter-wave plate is an optical device with an ordinary (slow) and an extraordinary (fast) axis. If light passes through, the polarization component which is parallel to the ordinary axis propagates slower than the other. So the slower component experiences a phase shift $\mathrm{e}^{i\delta}$. The quarter-wave plate can be described with the Jones matrix

$$M_{\mathrm{QWP}}^{(\mathrm{J})} = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{i\delta} \end{pmatrix}$$

with $\delta = \pi/2$. It can be used to transform linear polarization to circular polarization. Now the polarizer is parallely aligned to the incoming beam polarization. The fast axis and the transmission axis of the polarizer form an angle of 45°.
After passing this arrangement the light beam is reflected off a plane mirror and passes the arrangement in reversed order.

   i) Calculate the Jones vector for the transmitted light beam (before the reflection off the mirror). Explain how the additional phase of the slower component affects the propagation.

   ii) What happens to the reflected light? Calculate the final Jones vector.