Fundamentals of Optical Sciences

WS 2015/2016 **3 . Exercise** **02.11.2015**

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Prepare your answers for the exercise on 09.11.2015.

Exercise 1

Check whether the retarded potentials of an electric dipole oscillating along the z axis

$$\phi(\vec{r},t) \;=\; \frac{d_0\,\cos\theta}{4\pi\epsilon_0 r}\left(-\,\frac{\omega}{c}\,\sin\left[\omega\left(t-\frac{r}{c}\right)\right] + \frac{1}{r}\,\cos\left[\omega\left(t-\frac{r}{c}\right)\right]\right)$$

and

$$\vec{A}\,(\vec{r},t) \;=\; -\,\frac{\mu_0\,\omega\,d_0}{4\pi r}\,\sin[\,\omega(t-r/c)\,]\,\hat{e}_z \quad ,$$

fulfill the Lorenz gauge condition

$$\mathrm{div}\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t} \;=\; 0$$

not assuming that $r \gg c/\omega$. Here d_0 is the (constant) maximum dipole moment and $\hat{e}_z$ is the unit vector along the z axis.

Exercise 2

A radio tower rises to height h above flat horizontal ground. At the top is a magnetic dipole antenna, of radius b, with its axis vertical. FM station *SUN* broadcasts from this antenna at angular frequency ω, with a total radiated power P (that is averaged, of course, over a full radiation period). Neighbors have complained about problems they attribute to excessive radiation from the tower — interference with their stereo systems, garage doors opening and closing mysteriously, and a variety of suspicious medical problems. However, the engineer employed by the city council who measured the radiation level at the base of the tower found it to be well below the accepted standard. You have been hired by the Neighborhood Association to assess the engineer's report.

a) In terms of the variables given (not all of which may be relevant, of course), find the formula for the intensity of the radiation at ground level, at distance R from the base of the tower. You may assume that $a \ll c/\omega \ll h$. [*Note*: We are interested only in the *magnitude* of the radiation, not in its *direction* whereas when measurements are taken the detector will be aimed directly at the antenna.]

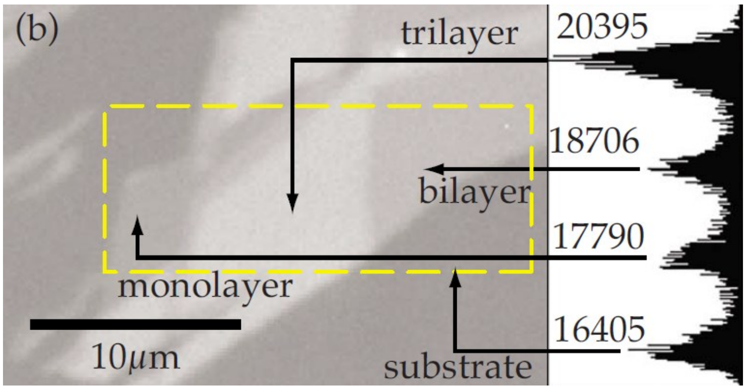
b) How far from the base of the tower *should* the engineer have made the measurement? What is the formula for the intensity at this location?

c) *SUN*'s actual power output is 35 kW, its frequency is 90 MHz, the antenna's radius is 6 cm and the height of the tower is 200 m. The city's radio-emission limit is $200\,\mu\text{W/cm}^2$. Is *SUN* in compliance?

Exercise 3

Exfoliated graphene on glass

Exfoliation of single graphene flakes from graphite is a wide-spread preparation technique. A simple tool to find and characterize even such thin layers is optical microscopy (see figure taken from DOI: 10.1063/1.3115026). Use the reflection-summation model to



calculate the contrast of a single graphene flake in an ordinary light microscope. Assume, that graphene was exfoliated onto perfectly smooth (and infinitely thick) glass. For simplicity neglect all illumination and detection angles other than normal incidence. Further, restrict the illuminating wavelength to only green light of 550 nm. For this wavelength consider a refractive index for glass of n_{glass}=1.5, for graphene of n_{gr}=2.5-i1.5 and for air of n_{air}=1, respectively. Find potential further relevant numbers at wikipedia.

Exercise 4

Consider a mirror waveguide with an air gap of $d = 2\mu m$. What is the cutoff frequency of such a waveguide? How many TE and TM modes does the waveguide support for a wavelength of $780nm$ and what are the corresponding bounce angles Θ_m? Derive the mode shape $E_x(y, z)$ of the m-th mode by calculating the interference pattern of the two corresponding propagating waves under the angles $\pm\Theta_m$.