Fundamentals of Optical Sciences

WS 2015/2016 **9 . Exercise** **14.12.2015**

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Prepare your answers for the exercise on 04.01.2016.

Exercise 1

Consider an ensemble of two-level atoms with the ground state $|\, g\,\rangle$ and excited state $|\, e\,\rangle$.

a) Give the density matrix for the following cases and draw the associated Bloch vector:

   i. All atoms are in the ground state $|\, g\,\rangle$.
   ii. The atoms from i) interact with a resonant π-pulse.
   iii. The atoms from i) interact with a resonant $\pi/2$-pulse.
   iv. One atom is in the ground state $|\, g\,\rangle$ and one in the excited state $|\, e\,\rangle$.

b) Which measurable properties are different in the cases iii) and iv)?

c) Now we include decay processes, e.g. spontaneous emission from state $|\, e\,\rangle$ to $|\, g\,\rangle$ with a decay constant γ. Therefore the optical Bloch equations have to be used. The equations

$$\dot{u} = \delta\, v - \frac{\gamma}{2}\, u$$
$$\dot{v} = -\delta\, u + \Omega_{ab} \cdot w - \frac{\gamma}{2}\, v$$
$$\dot{w} = -\Omega_{ab}\, v - \gamma(w-1)$$

with the auxiliary quantities

$$u = \rho_{12}\, \mathrm{e}^{-\mathrm{i}\delta t} + \rho_{21}\, \mathrm{e}^{\mathrm{i}\delta t}$$
$$v = -\mathrm{i}\, \left(\rho_{12}\, \mathrm{e}^{-\mathrm{i}\delta t} - \rho_{21}\, \mathrm{e}^{\mathrm{i}\delta t}\right)$$
$$w = \rho_{11} - \rho_{22}$$

describe the excitation of a two level system exposed to radiation close to resonance frequency, while accounting for spontaneous emission. Derive the steady-state equations using these equations. Write the solution obtained into vectorial form. Furthermore, derive an expression for the steady-state population in the $|\, b\,\rangle$-state. What happens in the limit of a strong driving field?

Exercise 2

In a Ramsey-type interference experiment two-level atoms interact with a $\pi/2$ pulse followed by a second $\pi/2$ pulse separated by a "delay" time T. During T the excited state accumulates a phase factor of $\mathrm{e}^{\mathrm{i}\delta T}$. Show that the transition probability for this type of experiment can be written as

$$P_e = 4\,\frac{\Omega_0^2}{\Omega^2}\sin^2\left(\frac{\Omega\tau}{2}\right)\left[\cos\left(\frac{\delta T}{2}\right)\cos\left(\frac{\Omega\,\tau}{2}\right)-\frac{\delta}{\Omega}\sin\left(\frac{\delta T}{2}\right)\sin\left(\frac{\Omega\,\tau}{2}\right)\right]^2$$

where τ is the interaction time of each of the two laser pulses of Rabi frequency Ω_0 and detuning δ. Use the general solutions

$$c_g(t) = \mathrm{e}^{\mathrm{i}\delta\tau/2}\left[c_g(0)\cos\left(\frac{1}{2}\Omega t\right)-\frac{\mathrm{i}}{\Omega}\left[\delta c_g(0)+\Omega_0 c_e(0)\right]\sin\left(\frac{1}{2}\Omega t\right)\right]$$
$$c_e(t) = \mathrm{e}^{\mathrm{i}\delta\tau/2}\left[c_e(0)\cos\left(\frac{1}{2}\Omega t\right)+\frac{\mathrm{i}}{\Omega}\left[\delta c_e(0)-\Omega_0 c_g(0)\right]\sin\left(\frac{1}{2}\Omega t\right)\right]$$

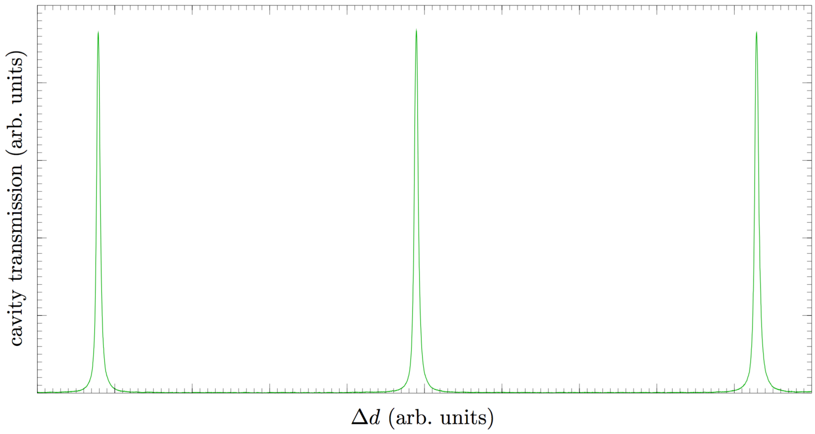
for the amplitudes. Assume that all atoms are initially in the ground state. The calculation is simplified, if the substitutions $\tau' = \Omega\,\tau/2$ and $T' = \delta\,T$ are used.
Reminder: The generalized Rabi frequency is $\Omega = \sqrt{\Omega_0^2+\delta^2}$.

Exercise 3

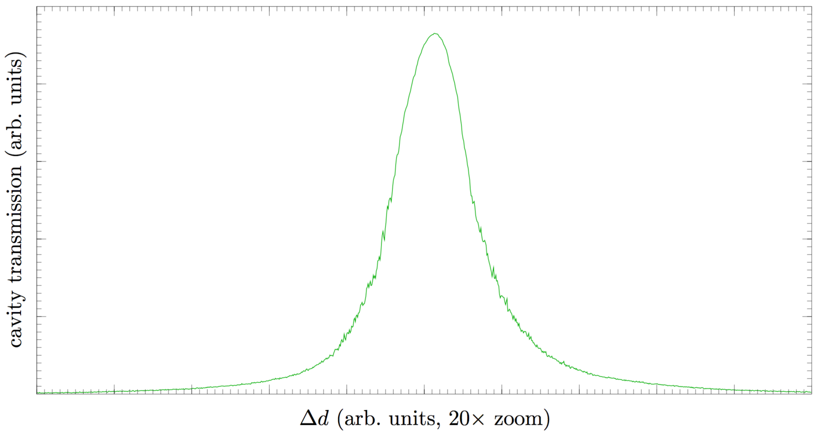
a) Consider a Fabry-Perot (planar) cavity of length 10 cm. Plot the cavity circulating intensity as a function of frequency for different amounts of loss in the cavity (for example, consider various mirror reflectivities $r = 0.2$, $r = 0.5$, $r = 0.8$, $r = 0.99$, corresponding to different finesse values). The plot should span several FSRs on the x axis. To see the different plots on the same scale, you can divide by the maximum intensity at each reflectivity. Assume an initial intensity of 1 W/m^2.

b) Name 3 other possible sources of loss in the cavity.

c) Consider a planar cavity operating at a nominal wavelength of 780 nm. The cavity length is 5 cm, but can be changed precisely (on a sub-wavelength scale) using a piezo stack attached to one of the mirrors. The cavity transmission is monitored as the cavity length d is scanned by translating the mirror. Below is the measured transmission spectrum:



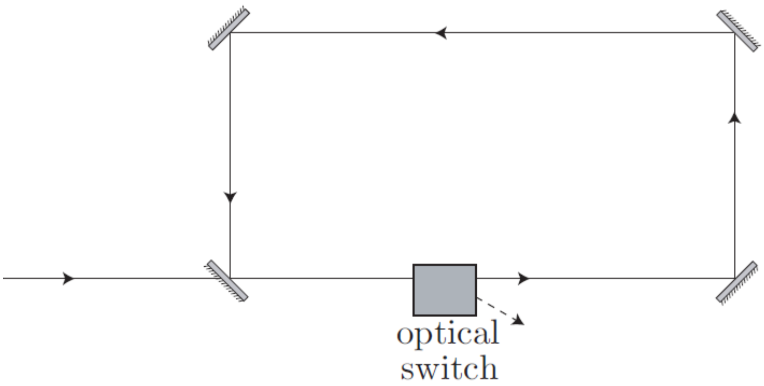
And here is the same set of data, with the horizontal axis zoomed by a factor of 20:



From these plots, find the FSR, finesse, resolution $\delta\nu_{\mathrm{FWHM}}$, and Q-factor of the cavity.

Exercise 4

One method for boosting the intensity of continuous-wave (cw) light is to use a cavity dumper, which is a ring cavity with an optical switch as shown.



The cavity dumper operates as follows: the input light is turned on, and the circulating light builds up in the cavity to a large intensity. Then the optical switch is activated, deflecting the circulating intensity out of the cavity.

a) What are the round-trip time and free spectral range? Assume the beam path is a rectangle of dimensions 12 × 20 cm. Also model the optical switch as a block of glass ($n = 1.5$) of length 2 cm.

b) What are the survival probability, finesse, and photon lifetime of the cavity? Assume an intensity reflection coefficient of 99.8% for all the mirrors and an intensity loss of 0.2% per pass due to the switch.

c) Suppose the input power is 1 W and that the circulating power is allowed to build up to its steady-state value. What are the duration and power of the output pulse when the switch is activated? Assume the switch couples all the light out of the cavity.

d) What is the maximum repetition rate of the cavity dumper if a 50 W output pulse is desired?