Blatt 10

1)

a) $s = v \cdot t \qquad t = \frac{s}{v} = \frac{1m}{5\frac{m}{s}} = \underline{\underline{0{,}2\,s}}$

b)

i) Wie weit hat sich die Maus bewegt?

$$\underbrace{v_{Maus} \cdot m_{Maus} + v_{Ball} \cdot m_{Ball}}_{\text{vor Abwurf}} = \underbrace{v'_{Maus} \cdot m_{Maus} + v'_{Ball} \cdot m_{Ball}}_{\text{nach Abwurf}} \qquad \text{(Impulserhaltg.)}$$

$v_{Maus} = v_{Ball} = 0$

$$\Rightarrow v'_{Maus} = -v'_{Ball} \frac{m_{Ball}}{m_{Maus}} = -5\frac{m}{s} \frac{2kg}{20kg} = -0{,}5\frac{m}{s}$$

$$s_{Maus} = v'_{Maus} \cdot t_{ges} = -0{,}5\frac{m}{s} \cdot 10s = \underline{-5m}$$

ii) Wie weit hat sich der Elefant bewegt?

In den ersten 0,2 s garnicht. Und danach:

$$\underbrace{v_E \cdot m_E + v'_{Ball} m_{Ball}}_{\text{während Flug}} = \underbrace{v'_E \cdot m_E + v''_{Ball} \cdot m_{Ball}}_{\text{nach Fangen}} \qquad \text{(Impulserhalt)}$$

$v_E = 0 \qquad\qquad v'_E = v''_{Ball}$

$$v'_{Ball} \cdot m_{Ball} = v'_E (m_E + m_{Ball})$$

$$v'_E = v'_{Ball} \frac{m_{Ball}}{m_E + m_{Ball}} = 5\frac{m}{s} \frac{2kg}{5kg + 2kg} = \frac{10}{7}\frac{m}{s}$$

Bewegung vom Elefant für $t > 0{,}2s$

$$s_E = s_0 + v'_E t = 1m + \frac{10}{7}\frac{m}{s} \cdot 9{,}8s = \underline{15m}$$

$\Rightarrow$ Abstand d nach 10s

$$d = s_E - s_{Maus} = 15m + 5m = \underline{\underline{20m}}$$

c) $$\underbrace{v'_E (m_E + m_{Ball})}_{\text{vor Abwurf}} = \underbrace{v''_E m_E + p'''_{Ball}}_{\text{nach Abwurf}} \qquad \text{(Impulserhaltg.)}$$

$$v'_E (m_E + m_{Ball}) \overset{b)}{=} v'_{Ball} \cdot m_{Ball} = v''_E m_E + p'''_{Ball}$$

$$p'''_{Ball} = v'_{Ball} \cdot m_{Ball} - v''_E m_E = 5\frac{m}{s} \cdot 2kg - (-5\frac{m}{s}) \cdot 5kg = \underline{\underline{35\,kg\frac{m}{s}}}$$

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2) a) $\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{\vec{a}_0}{2} t^2$

$$= \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} v_{x0} \\ v_{y0} \end{pmatrix} t + \begin{pmatrix} a_{0x} \\ a_{0y} \end{pmatrix} \frac{t^2}{2}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + v_0 t \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} + \begin{pmatrix} 0 \\ -g \end{pmatrix} \frac{t^2}{2}$$

$x(t) = v_0 t \cos\alpha$

$y(t) = v_0 t \sin\alpha - \frac{g}{2} t^2$

b) Zeitpunkt der maximalen Höhe t_{max}

$\dot{y}(t) = v_0 \sin\alpha - g t$

$0 = \dot{y}(t_{max}) = v_0 \sin\alpha - g t_{max} \longrightarrow t_{max} = \frac{v_0 \sin\alpha}{g}$

$$y(t_{max}) = v_0 \frac{v_0 \sin\alpha}{g} \sin\alpha - \frac{g}{2} \frac{v_0^2 \sin^2\alpha}{g^2} = \frac{v_0^2 \sin^2\alpha}{g} - \frac{v_0^2 \sin^2\alpha}{2g} = \frac{v_0^2 \sin^2\alpha}{2g}$$

$$= \frac{(15\frac{m}{s})^2 \sin^2(20^\circ)}{2 \cdot 10 \frac{m}{s^2}} \approx \underline{\underline{1{,}32\,m}}$$

c) Zeitpunkt Pfeil bei Baum t_{Baum}

$s = x(t_{Baum}) = v_0 t_{Baum} \cos\alpha \longrightarrow t_{Baum} = \frac{s}{v_0 \cos\alpha}$

Höhe der Pfeils am Baum

$$y_p(t_{Baum}) = v_0 \frac{s}{v_0 \cos\alpha} \sin\alpha - \frac{g}{2} \frac{s^2}{v_0^2 \cos^2\alpha} = \underline{s \tan\alpha - \frac{g}{2} \frac{s^2}{v_0^2 \cos^2\alpha}} \quad \text{I}$$

Bewegung von Leo $y_L = h - \frac{g}{2} t^2$

Position von Leo bei t_{Baum}

$$y_L(t_{Baum}) = h - \frac{g}{2} t_{Baum}^2 = h - \frac{g}{2} \frac{s^2}{v_0^2 \cos^2\alpha} \quad \text{II}$$

Pfeil trifft, wenn $y_L(t_{Baum}) = y_p(t_{Baum}) \rightarrow y_L(t_{Baum}) - y_p(t_{Baum}) = 0$

Test:

$$y_L(t_{Baum}) - y_p(t_{Baum}) = h - \frac{g}{2} \frac{s^2}{v_0^2 \cos^2\alpha} - s \tan\alpha + \frac{g}{2} \frac{s^2}{v_0^2 \cos^2\alpha}$$

$$= h - s \tan\alpha = 3{,}6\,m - 10\,m \tan(20^\circ) = 3{,}6\,m - 3{,}64\,m \approx 0$$

$\Rightarrow$ Der Pfeil trifft.

Blatt 10

3)

a) $E_{Anfang} = E_{pot} + E_{kin} = mgh_0 + \frac{m}{2} v_0^2$

$E_A = E_{kin} = \frac{m}{2} v_r^2$

$E_A = E_{Anfang}$

$\frac{m}{2} v_r^2 = mgh_0 + \frac{m}{2} v_0^2$

$v_r = \sqrt{2gh_0 + v_0^2} = \sqrt{2 \cdot 10\frac{m}{s^2} \cdot 10m + (10\frac{m}{s})^2} = \sqrt{300\frac{m^2}{s^2}} \approx \underline{\underline{17{,}3\frac{m}{s}}}$

b) $E_{Feder} = \frac{D}{2} x^2$

$E_{Anfang} = E_{Feder}$

$mgh_0 + \frac{m}{2} v_0^2 = \frac{D}{2} x^2$

$x = \sqrt{\frac{2mgh_0}{D} + \frac{m}{D} v_0^2} = \sqrt{\frac{m}{D}} \sqrt{2gh_0 + v_0^2} = \sqrt{\frac{50kg\, s^2 m}{240 \cdot 10^3 kg\, m}} \sqrt{300\frac{m^2}{s^2}} \approx \underline{\underline{0{,}25m}}$

c) $\Delta E = E_{Anfang} - E_{Ende} = mgh_0 + \frac{m}{2} v_0^2 - mgh_1$

$= mg(h_0 - h_1) + \frac{m}{2} v_0^2$

$= 50kg \cdot 10\frac{m}{s^2}(10m - 8m) + \frac{50kg}{2}(10\frac{m}{s})^2$

$= 3500 \frac{kg\, m^2}{s^2}$

$= \underline{\underline{3{,}5kJ}}$

Blatt 10

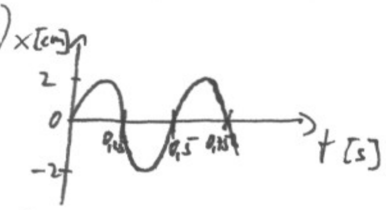
4)

a) $x(t) = A \sin(\omega t + \varphi_0)$

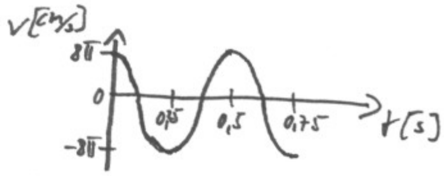
$v(t) = \dot{x}(t) = A\omega \cos(\omega t + \varphi_0)$

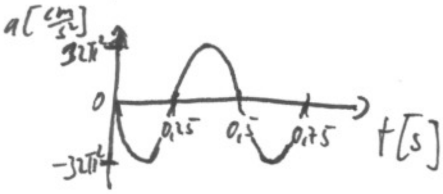
$a(t) = \ddot{x}(t) = -A\omega^2 \sin(\omega t + \varphi_0)$

b)



$\omega = \frac{2\pi}{T}$ $\quad T = \frac{2\pi}{\omega} = \frac{2\pi\, s}{4\pi} = 0{,}5\, s$





c) siehe b) $\underline{\underline{T = 0{,}5\, s}}$

$\omega = 2\pi f$ $\quad f = \frac{\omega}{2\pi} = \frac{4\pi}{2\pi} s^{-1} = \underline{\underline{2\, s^{-1}}}$

Blatt 10

5)

a) Drehimpulserhaltg.: ausgestrecktes Bein angezogenes Bein

$$L_1 = L_2$$

$$\theta_1 \omega_1 = \theta_2 \omega_2$$

$$\omega_2 = \frac{\theta_1}{\theta_2}\omega_1 \qquad \left(\omega = \frac{2\pi}{T}\right)$$

$$\frac{2\pi}{T_2} = \frac{\theta_1}{\theta_2}\omega_1$$

$$T_2 = \frac{2\pi}{\omega_1}\frac{\theta_2}{\theta_1}$$

$$= \frac{2\pi s}{6}\frac{2\cancel{kg\,m^2}}{6\cancel{kg\,m^2}} = \frac{\pi}{9}s \approx \underline{\underline{0{,}35s}}$$

b) $E_{rot} = \frac{\theta}{2}\omega^2$

$$\frac{E^1_{rot}}{E^2_{rot}} = \frac{\theta_1 \omega_1^2 \cancel{\frac{1}{2}}}{\theta_2 \omega_2^2 \cancel{\frac{1}{2}}} \overset{a)}{=} \frac{\theta_1 \omega_1^2}{\theta_2 \frac{\theta_1^2}{\theta_2^2}\omega_1^2} = \frac{\theta_2}{\theta_1} = \frac{2\,kg\,m^2}{6\,kg\,m^2} = \underline{\underline{\frac{1}{3}}}$$

c) die zusätzliche Energie entspricht der Arbeit, die beim Anziehen des Beins verrichtet wird

d) $\theta = \frac{m}{2}R^2 \qquad m = \frac{2\theta}{R^2} = \frac{2 \cdot 2\,kg\,m^2}{(0{,}3m)^2} = \underline{\underline{44{,}44kg}}$