Problem Set 3, Introduction to Laser Physics, WS 10/11   11/01/11

Topic: rate equations

Please answer the following questions until 01- 11-2011

Problem 7 – problem definition

Introduction: Consider the 3-level scheme 2.3.1-1. Under which condition can the upper pump level 3 be eliminated, so that the 3-level system can be replaced by an effective 2-level system?

Problem 7.1

Write down a complete set of rate equations for the 3-level system. Assume that the states are not degenerate and that no laser field is present (which can be ensured for example by blocking one of the mirrors in a laser setup).

Problem 7.2

Specialize the rate equations for the steady state. To simplify the future discussion please replace the corresponding quantities by the expressions given below. Explain, why these replacements are reasonable from the physics point of view. Explain the physical meaning of these quantities.

- give all rates in units of γ_{21}, e.g. $g_{32} = \gamma_{32}/\gamma_{21}$

- give all population densities in units of the absolute population density n_{tot},
  e.g. $n_1' = n_1/n_{tot}$

- replace all relative populations n'_1, n'_2, and n'_3 by the following set of populations: relative inversion n', relative total population n'_0 of level 1 and 2, and relative population n'_3 of level 3.

- Replace the relative decay rates g_{32} and g_{31} by

$$g_{32} = \eta \cdot g_3, \quad g_{31} = (1-\eta) \cdot g_3$$

  What is the physical meaning of η und g_3?

Problem 7.3

Solve the reformulated rate equations for steady state.

Problem 7.4

Under which condition can the population in level 3 be neglected?
Hint: a series expansion might be useful.

Topic: rate equations

Problem 7.5
Under which condition is inversion achieved?

Hint: Calculate the inversion. To simplify the math, introduce the following quantities:

$$x = w_{13}/g_3 , \ \ y = \eta \cdot g_3$$

What is the physical meaning of x and y?
Formulate the inversion condition in terms of x and y.
How does this inversion condition look like if the approximation is applied which has already been used to eliminate level 3?