































Dielectric Formulation of Energy Loss

The dielectric formulation of energy losses considers a continuum of varying (complex) dielectric constant, instead of individual atomic orbitals.

Double differential cross section $(q^2 = k_0^2 (\theta^2 + \theta_E^2))$

$$\frac{d^2\sigma}{dEd\Omega} = \frac{const}{q^2} Im\left(-\frac{1}{\varepsilon(E,q)}\right) = \frac{const}{\theta^2 + \theta_E^2} Im\left(-\frac{1}{\varepsilon(E,q)}\right)$$

Kramers Kronig Relation relates real and imaginary part of the dielectric function. The full complex $\varepsilon = \varepsilon_1 + i\varepsilon_2$ can therefore be obtained from a single EEL spectrum ($E = \hbar \omega$)

$$Re\left(\frac{1}{\varepsilon(\omega)}\right) - I = \frac{1}{\pi} \mathbb{P} \int_{-\infty}^{\infty} Im\left(\frac{1}{\varepsilon(\omega')}\right) \frac{1}{\omega' - \omega} d\omega'$$

$$\int_{-\infty}^{\infty} \frac{Re\left(\frac{1}{\varepsilon}\right)}{\left(Re\left(\frac{1}{\varepsilon}\right)^{2} + \left(Im\left(\frac{1}{\varepsilon}\right)\right)^{2}}\right)} \frac{\varepsilon_{2}}{\varepsilon_{2}} = \frac{Im\left(\frac{1}{\varepsilon}\right)}{\left(Re\left(\frac{1}{\varepsilon}\right)^{2} + \left(Im\left(\frac{1}{\varepsilon}\right)\right)^{2}}\right)}$$
Max-Planck Institut für Metallforschung Page: 17 Universität Stuttgart

Models of the Dielectric Function

<u>Drude Model</u>: Valid for Metals and Semiconductors. Assumes the existence of a single plasmon resonance frequency ω_p and a damping coefficient τ (small for metals). ω_p depends on the valence electron density n_e and the effective electron mass m.

$$\varepsilon(\omega) = \varepsilon_1 + i \varepsilon_2 = 1 - \frac{\omega_p^2}{\omega^2 + 1/\tau^2} + \frac{i1/\tau \omega_p^2}{\omega(\omega^2 + 1/\tau^2)} \qquad \qquad \omega_p = \left(\frac{n e^2}{\varepsilon_0 m}\right)$$

<u>Lorentz Model</u>: Assumes an ensemble of many different harmonic oscillators representing possible interband transitions with energies equal to the oscillators eigenfrequency ω_n . Commonly only one such harmonic oscillator is assumed:

 $\varepsilon_n = 1 + \frac{ne^2}{m \varepsilon_0} \frac{1}{\omega_n^2 - \omega^2 + i\frac{\omega}{\tau}}$ Max-Planck Institut für Metallforschung Page: 18 Universität Stuttgart































































