







A fast electron in a periodic potential $V(\mathbf{r})$

(Bloch wave method: Hans Bethe, 1928)

The relativistic Schrödinger equation (Klein Gordon equation) for a fast electron:

$$\nabla^2 \Psi(\vec{r}) + \frac{2m|e|}{\hbar^2} \left[E + V(\vec{r}) \right] \Psi(\vec{r}) = 0$$

Expanding $\Psi(r)$ using **Bloch's theorem**, the Laplace operation on the wave function can be written in reciprocal space as:

$$\nabla^2 \Psi = -4\pi^2 \sum_j \Psi_j \sum_{\vec{g}} C_{\vec{g}}^{(j)} (\vec{k}^{(j)} + \vec{g})^2 \exp(2\pi i (\vec{k}^{(j)} + \vec{g}) \cdot \vec{r})$$

A somewhat lengthy derivation shows that the $C_g^{(j)}$ -coefficients must also be the elements of the eigenvector matrix of the so called **structure factor matrix** and $(k_z^{(j)} - K)$ its eigenvalues, but also, that given these, the reciprocal-space wave function behind the crystal (z=t) can be obtained by that in front of it (z=0) according to: $\vec{x} \cdot (\vec{x} - t) = c_1 [\vec{x} \cdot (\vec{x} - t)]$

$$\tilde{\Psi}(\vec{k}_t, t) = C \left[\exp(2\pi i k_z^{(j)} t) \right]_D C^{-1} \tilde{\Psi}(\vec{k}_t, 0)$$

Scattering matrix $S(k_t, \lambda, t)$

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